U.S. Import Price Indices Based on an Exact Index Formula:

For Selected Import Product Groups

by

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Abstract

It is believed that changes in the varieties of an imported product might have effects on the import price index for the product that are similar to the effects of new goods on the cost of living. Recently, a new index number formula that incorporates the effects of new goods has been suggested by Nahm (1998a). The index number formula is a known function of available values, and it is exact for the constant-elasticity-of-substitution (CES) preference ordering even when there exist new and disappeared goods. The formula has been applied to each of eight selected U.S. import products to compute each product’s import price index. As each product has several varieties and the set of varieties changes over time, a price index for the product should reflect changes in the choice set. The new index formula is designed to capture such effects. The results show that the indices based on the new index formula closely trace those based on a sophisticated econometric model.

Keywords: price index, exact index, CES function

JEL Classification: C43
1. Introduction

It has been more than six years since the *Boskin Commission* reported in December 1996 that the U.S. *Consumer Price Index* (CPI)-based inflation rate overstates the true rate of change in the *cost of living* (COL) by 1.1 percentage points per year (Boskin et al. [1996]). Out of the 1.1 percentage point bias, 0.4 percentage points was due to the wrong index-number formula used for the upper and lower level aggregation, while 0.6 percentage points was attributable to ignoring or wrong treatment of new products and quality changes. The remaining 0.1 percentage point was reported as due to ignoring the effect of outlet substitution.

Since the report, the U.S. *Bureau of Labor Statistics* (BLS) has implemented many of the Commission’s recommendations in an effort to reduce the bias in the CPI as a measure of the COL. The changes made to the way the U.S. CPI is computed include employing a geometric-mean index number formula for lower-level aggregation, more rapid changes in upper-level weights, employing the hedonic-regression technique to account for the effects of quality changes on computer prices and television prices, and other changes in the procedure of data collection and sample rotation; see Gordon (1999). Gordon estimates that the changes adopted by the BLS might have eliminated about half of the 1.1 percentage point bias reported by the Commission. Most of the 0.4 percentage point substitution bias and about 0.2 to 0.3 percentage point quality change and new goods bias is expected to have disappeared in the new CPI.

However, still remaining in the CPI is a significant part of quality change and new goods bias and outlet substitution bias. Especially, the effect of having new products available in the marketplace on the COL has yet to be accounted for. While including new products more rapidly in the weights may alleviate potential bias, it does not fully account for the effect on the COL of having new products available. Having a different choice set itself has an effect on the COL as consumers’ substitution opportunity changes.

Feenstra (1994) has noted similar effects in the import-price indices for various products that are imported into the United States. Feenstra was concerned with the effect of changes in the number
of varieties of a product imported into the United States on the import-price indices of the product. As the group of countries that export a product to the U.S. varies from year to year, price indices for an imported product should account for the effect of changing choice sets.

To aggregate the prices of several varieties of each imported product, he introduced a novel index number formula that is exact when preferences are represented by a constant-elasticity-of-substitution (CES) utility or expenditure function.\(^1\) There had been an index number formula exact for the CES preference ordering before Feenstra introduced his index, but the former is valid only if there is no change in the choice sets, namely, when there is no "new" or "disappeared" goods. Feenstra's index overcomes the problem and it takes into account of the effects of new and disappeared goods on a price aggregate. A disadvantage of Feenstra's index, however, is that it has an unknown constant in it, and hence it has to be estimated separately. Recently, Nahm (1998a) has introduced an index number formula that replaces the unknown constant in Feenstra's index with a known function of available values. It should be attractive in particular to statistics agencies, such as the U.S. BLS and the Australian Bureau of Statistics (ABS), because it provides a routine and consistent method to compute price aggregates under the circumstance where new products/varieties are introduced and some products disappear.

The objective of this paper is to apply the new index number formula to the import-price data analysed by Feenstra (1994), and to examine how it performs with the real data compared with Feenstra's formula.

2. Exact Price Index Formulas Under the CES Preference Ordering

For the expositions in this section, suppose that we are interested in the price index of a group of varieties available in period 1 in comparison with the aggregate price of the varieties available in period 0. Let \(I\) be the set of varieties available in period \(t\) for \(t = 0,1\), and set \(I\) be the set of

\(^1\) An index number formula, which is a known function of observable values of prices and quantities, is said to be exact for an aggregator function if the index is equal to the true index implied by the aggregator function when the observed prices and quantities are optimal; see Diewert (1976).
varieties that are available in the both periods. Then, set I is the intersection set of $I^0$ and $I^1$. It is assumed that there are at least two varieties in set I.

When preferences are represented by the CES expenditure function, $E(p,u)$, where $p$ is the price vector and $u$ is the level of utility, the true price index is given by $E(p^1,u,I^1)/E(p^0,u,I^0)$, with $p^1$ and $p^0$ being the price vectors in periods 1 and 0 respectively, and $u$ being the reference utility level. As Nahm (1998b) points out, the overall price index can be decomposed into three components:

$$
\frac{E(p^1,u,I^1)}{E(p^0,u,I^0)} = \frac{E(p^1,u,I^1)}{E(p^1,u,I)} \cdot \frac{E(p^1,u,I)}{E(p^0,u,I)} \cdot \frac{E(p^0,u,I)}{E(p^0,u,I^0)}.
$$

The middle part in the right–hand side measures the effect of price changes for those products that are available in both periods. The first term in the right–hand side represents the effect of the introduction of new products on the price index, while the third term represents the effect of disappeared products.

Vartia (1974, 1976) and Sato (1976) introduced the following index number formula that is exact for the middle component of (1).³

$$
P_v(p^0,p^1,q^0,q^1,I) \equiv \prod_{i \in I} \left( \frac{p^1_i}{p^0_i} \right)^{w_i(I)}
$$

where

$$
w_i(I) \equiv \frac{[s_i^1(I) - s_i^0(I)]/[\ln s_i^1(I) - \ln s_i^0(I)]}{\sum_{j=1}^{n_{I^1}} [(s_j^1(I) - s_j^0(I))]/[\ln s_j^1(I) - \ln s_j^0(I)]}, \text{ and}
$$

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² The CES functional form has special properties that enable this type of separability. See Nahm (1998a, Section 1) for more details.

³ This index formula is known in the literature as the Vatira II index.
\[ s_i^t(I) \equiv \frac{p_i^t q_i^t}{\sum_{j \in I} p_j^t q_j^t}, \] which is the budget share of variety \( i \) in set \( I \) in period \( t \).

Hence, the Vartia index given above is exact for the CES expenditure function when all varieties are available in both periods 0 and 1. However, the formula is not usable when there are new or disappeared varieties. Feenstra (1994) introduces the following formula that overcomes this problem of the Vartia index.

\[ P_F(p^0, p^1, q^0, q^1, I^0, I^1; \sigma) \equiv P_V(p^0, p^1, q^0, q^1, I) \left( \frac{\lambda^I(I)}{\lambda^0(I)} \right)^{1/(\sigma - 1)} \]

where

\[ \lambda^I(I) \equiv \frac{\sum_{i \in I} p_i^t q_i^t}{\sum_{j \in I} p_j^t q_j^t}. \]

For the CES expenditure function \( E(.) \), the above formula is exact for the true overall price index, \( E(p^1, u, I^1)/E(p^0, u, I^0) \). That is, Feenstra’s index is exact for the CES function even if some varieties are not available in one of the two periods. A disadvantage of the above index is, however, that it has an unknown constant \( \sigma \) and thus it has to be estimated separately. The index number formula introduced by Nahm (1998a) replaces the \( \sigma \) with a known function of available values and hence overcomes the disadvantage of Feenstra’s index.

\[ P_N(p^0, p^1, q^0, q^1, I^0, I^1) = P_V(p^0, p^1, q^0, q^1, I) \left[ \frac{\lambda^I(I)}{\lambda^0(I)} \right]^\beta, \] where

\[ \beta = \frac{y_i}{x_i} \text{ for all } i \in I^\circ \equiv \{ i : i \in I \text{ and } s_i^0(I) \neq s_i^1(I) \} \]

with \( y_i \) and \( x_i \) defined by \( y_i \equiv \ln P_V(p^0, p^1, q^0, q^1, I) - \ln(p_i^1/p_i^0) \), and \( x_i \equiv \ln s_i^1(I) - \ln s_i^0(I) \).

The above index is exact for the CES function even if not all varieties are available in the both periods. The \( \beta \) is a known function of available prices and quantities. Nahm (1998a) also shows
that the effect of new varieties can be exactly measured by \([\lambda^1(I)]^\beta\), and the effect of disappeared varieties by \([\lambda^0(I)]^{-\beta}\).

Any functional form for \(\beta\) would be valid as long as equation (7) is satisfied. If the prices and quantities of all the varieties in set I are optimal, any \((y_i/x_i)\) for \(i \in I^*\) will render the new index exactly equal to the true aggregate price index. In reality, however, prices and quantities include random factors and hence the theoretical relationships would barely hold exactly. Hence, averaging \((y_i/x_i)\) over \(i \in I\) in some way would be desirable. In the next section, the following three alternative formulas for \(\beta\) will be considered:

\[
\beta_1 \equiv \frac{1}{n} \sum_{i=1}^{n} \left( \frac{y_i}{x_i} \right), \quad \beta_2 \equiv \frac{\sum_{i \in I} y_i}{\sum_{i \in I} x_i}, \quad \text{and} \quad \beta_3 \equiv \frac{\sum_{i \in I} (y_ix_i)}{\sum_{i \in I} x_i^2}.
\]

Note that all these three formulas are equal to \(\beta\) when all values are optimal. Thus, the new index is \textit{exact} for the CES function whichever formula is used. The first formula, \(\beta_1\), is a mean ratio. In the context of the equation, \(y_i = \beta x_i\), \(\beta_2\) is an instrumental-variable (IV) formula with a constant as the instrument and \(\beta_3\) is the ordinary least-squares (OLS) formula. The indices based on these three formulas of \(\beta\) will not be necessarily identical to each other when the prices and quantities contain random factors.

3. An Application of the New Formula to the U.S. Import Prices

Feenstra (1994) uses the formula defined by (4) to compute yearly price indices of each of eight import-product groups for the period from 1965 to 1987. The new index number formula, (6), with alternative formulas for \(\beta\) given in (8), has been applied to the same data.

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4 The eight product groups are Athletic Shoes, Cotton Knit Shirts, Stainless Steel Bars, Carbon Steel Sheets, Color TV Receivers, Portable Typewriters, Gold Bullion and Silver Bullion. Exceptions to the sample period are Color TV Receivers (71–87), Gold Bullion (78–87), and Silver Bullion (69–87).
For each of the eight products, each year's price index is computed using the two-period price index formula defined in the previous section, and then a time series of price indices over the whole sample period with the first year in the sample as the base period is constructed using the chain principle:

\[
P_a(p^1,\ldots,p^t,q^1,\ldots,q^t,I^1,\ldots,I^t) = \prod_{t=2}^{t} P_a(p^{t-1},p^t,q^{t-1},q^t,I^{t-1},I^t)
\]

\[P_a(p^1,p^1,q^1,q^1,I^1) = 1 \quad \text{for } a = \text{Vartia, Feenstra, and New}; \quad t = \text{first year},\ldots,87.
\]

In computing each two-period index, two alternative sets for I are considered:

\[I(t)_1 \equiv \{i: i \in I^{t-1} \text{ and } i \in I^t\}, \text{ and}
\]

\[I(t)_2 \equiv \{i: i \in I(t)_1 \text{ and } i \in \text{industrial countries}\} \quad \text{for } t = \text{second year},\ldots,87.
\]

The above two sets are identical to the first two sets defined by (13a) and (13b) in Feenstra (1994), respectively.

The law of demand dictates that the $\beta$ should be positive as it is the negative of the ratio between the log of a change in the price of a product relative to the average price change and the corresponding change in the log of budget share of the same product. This condition coincides with the requirement for the elasticity of substitution to be greater than unity. As the law of demand could be violated in the unrestricted data (UR) set, two more restricted data sets in addition to the UR data set are also considered:

\[\text{(12) Truncated data (TD) set: observation } i \text{ is excluded in computing } \beta \text{ if } y_i x_i \leq 0, \text{ and}
\]

\[\text{(13) Censored data (CD) set: } y_i = 0 \text{ if } y_i x_i \leq 0, \text{ and } y_i \text{ is unchanged otherwise.}
\]

Note, however, that neither TD nor CD set guarantees a positive value of an IV or OLS estimate for $\beta$ even though they increase the probability to have a positive value significantly.
Table 1: Estimates of the elasticity of substitution (σ)*

<table>
<thead>
<tr>
<th>Estimator</th>
<th></th>
<th>Mean Ratio</th>
<th></th>
<th>IV</th>
<th></th>
<th>OLS</th>
<th></th>
<th>IOLS+</th>
<th></th>
<th>Feenstra's</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Data set</td>
<td>UR</td>
<td>TD</td>
<td>CD</td>
<td>UR</td>
<td>TD</td>
<td>CD</td>
<td>UR</td>
<td>TD</td>
<td>CD</td>
</tr>
<tr>
<td>Shoes</td>
<td>190/413#</td>
<td>-2.60</td>
<td>2.19</td>
<td>3.60</td>
<td>-12.65</td>
<td>5.27</td>
<td>4.54</td>
<td>-20.85</td>
<td>5.62</td>
<td>14.01</td>
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<tr>
<td></td>
<td>94/213</td>
<td>-12.86</td>
<td>1.77</td>
<td>2.73</td>
<td>-0.08</td>
<td>-0.08</td>
<td>4.54</td>
<td>-12.46</td>
<td>6.64</td>
<td>16.65</td>
</tr>
<tr>
<td>Shirts</td>
<td>306/634</td>
<td>9.39</td>
<td>1.54</td>
<td>2.13</td>
<td>2.76</td>
<td>3.24</td>
<td>3.65</td>
<td>-86.50</td>
<td>5.57</td>
<td>10.48</td>
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<tr>
<td></td>
<td>108/247</td>
<td>-24.31</td>
<td>1.59</td>
<td>2.34</td>
<td>2.15</td>
<td>2.98</td>
<td>2.42</td>
<td>3770.7</td>
<td>4.78</td>
<td>8.67</td>
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<tr>
<td>St. Rars</td>
<td>94/216</td>
<td>3.15</td>
<td>1.38</td>
<td>1.87</td>
<td>11.38</td>
<td>8.36</td>
<td>8.67</td>
<td>75.67</td>
<td>6.07</td>
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<td>2.78</td>
<td>11.29</td>
<td>6.07</td>
<td>6.92</td>
<td>20.93</td>
<td>5.29</td>
<td>8.53</td>
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<tr>
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<td>2.63</td>
<td>4.00</td>
<td>6.58</td>
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<td>5.80</td>
<td>4.59</td>
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<td>4.93</td>
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<tr>
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<td>11.41</td>
<td>1.74</td>
<td>2.28</td>
<td>-10.76</td>
<td>4.36</td>
<td>7.17</td>
<td>-40.39</td>
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<td>10.15</td>
</tr>
<tr>
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<td>36/58</td>
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<td>2.18</td>
<td>2.91</td>
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<td>6.31</td>
<td>13.52</td>
<td>52.89</td>
<td>4.93</td>
<td>9.22</td>
</tr>
<tr>
<td>Typewrit.</td>
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<td>-16.91</td>
<td>1.59</td>
<td>2.51</td>
<td>-0.86</td>
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<td></td>
<td>74/189</td>
<td>1.85</td>
<td>1.22</td>
<td>1.55</td>
<td>-1.70</td>
<td>1.54</td>
<td>3.75</td>
<td>-4.46</td>
<td>5.67</td>
<td>14.16</td>
</tr>
<tr>
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<td>12.30</td>
<td>-0.25</td>
<td>9.07</td>
<td>-1.15</td>
<td>334.30</td>
<td>18.40</td>
<td>36.62</td>
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<tr>
<td></td>
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<td>8.10</td>
<td>13.54</td>
<td>7.35</td>
<td>14.09</td>
<td>7.88</td>
<td>61.86</td>
<td>12.73</td>
<td>30.15</td>
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<tr>
<td>Silver</td>
<td>104/223</td>
<td>14.08</td>
<td>2.97</td>
<td>5.22</td>
<td>-41.13</td>
<td>15.26</td>
<td>11.70</td>
<td>-91.16</td>
<td>18.00</td>
<td>46.22</td>
</tr>
</tbody>
</table>

*: The values in columns 2–10 are computed using pooled data, while those in column 11 are the averages of the values that are computed using the individual cross-sectional data. The values in the upper rows are the estimates based on set I(t)1 while those in the lower rows are the estimates based on set I(t)2.

#: The number of observations used for the estimation. The denominator is the number of observations in the UR and CD data sets while the numerator is the number of observations in the TD data set.

+: Estimates of σ are obtained by using the average of the (T−1) estimates of β for the (T−1) pairs of consecutive time periods (where T denotes the time series). The periods with a zero or negative estimate of β are excluded in computing the averages.

The CES functional form implies that σ (and hence β) is constant over time as well as across pairs of products. To meet this requirement and also to make the estimates comparable with those by Feenstra, the observations in I(t) are pooled over all time periods for the estimation of β so that the same estimate of β is applied to all time periods in computing price indices. For the OLS formula, an intercept term is included for econometric reasons so that yi = α + βxi. Theoretically, the α should be zero. If its estimate were significantly different from zero when computed using the values with random factors, it would represent a systematic bias in the Vartia index as a measure of the average price change in the presence of random factors.

5 The main purpose of pooling data is to make the new index series comparable with the Feenstra’s series as he used the same estimate of σ for all time periods. The new index series without this restriction are also computed below.
Columns 2-10 in Table 1 report the estimates of $\sigma (= 1/\beta + 1)$ that have been obtained by applying the three formulas of $\beta$ to the pooled data sets. The value in the upper row of each cell is an estimate based on $I(t)_1$ while the value in the lower row is an estimate based on $I(t)_2$. The numbers of observations used for the estimation are reported in the first column. For example, in the case of athletic shoes, there are 413 observations in each of the pooled UR and CD sets and 190 observations in the pooled TD set when the first definition of the intersection set, $I(t)_1$, is used. When $I(t)_2$ is used, the numbers reduce to 213 and 94 respectively.

For all three formulas, the pooled UR estimation results are highly unstable. In many cases, the estimates for $\sigma$ are less than unity against the requirement. The estimates are also sensitive to the change of the intersection set from $I(t)_1$ to $I(t)_2$. On the other hand, the estimates based on the pooled TD and CD sets are much more stable and much less sensitive to the change of the intersection set. The mean-ratio estimates are smaller than the corresponding OLS estimates and those obtained by Feenstra (reported in the last column) for all products. This implies that the estimates for $\beta$ are larger than their OLS counterparts and those implicit in Feenstra (1994). If both $y_i$ and $x_i$ were normally distributed, the density function of the mean-ratio estimator will have a Cauchy component in it and hence its mean and variance will be undefined; see, for example, Mood, Graybill and Boes (1974, Chapter 5).

Some of the IV estimates are less than one, implying that the corresponding estimates for $\beta$ are negative even with the TD and CD sets. This estimator would be appropriate if the values were measured with errors because it would provide consistent estimates even under such a circumstance. However, the variance of the IV estimator is expected to be large because of the poor quality of a constant as an instrument. The large variance of the estimator appears to be the main reason for the unreliable estimates in the present case.

The OLS estimates with the pooled TD and CD sets are relatively stable and close to the corresponding estimates by Feenstra although larger than Feenstra's in most cases. When the data
set is truncated or censored as described above, the OLS estimator for $\beta$ ($>0$) is asymptotically biased upward with the truncation causing a larger bias. With the CD set, it can be shown that the OLS estimator converges to the following value as the number of observations increases:

$$
\beta \left[ 1 + \frac{1}{\pi} \left( \frac{1}{A} - \frac{\pi}{2} + \tan^{-1} A \right) \right] \quad \text{for } 0 < A < 1,
$$

where $A \equiv \beta \sigma_x/\sigma_u$, $\sigma_x$ is the standard deviation of $x$, and $\sigma_u$ is the standard deviation of the random error term in $Y_i = \alpha + \beta x_i + u_i$. It can be also shown that the second term in the square bracket is a positive value for $A > 0$, and it converges to zero as $A$ increases.

In the absence of measurement errors in the prices and quantities, the TD and CD-OLS estimates for $\beta$ would be larger than the true values and hence the estimates for $\sigma$ would be smaller than their true values. As expected, the TD-OLS estimates for $\sigma$ are smaller than their CD-OLS counterparts as the TD-OLS estimates for $\beta$ are larger than the corresponding CD-OLS estimates for $\beta$. However, what is unexpected is that in most cases the CD- and TD-OLS estimates for $\sigma$ are larger than their counterparts in Feenstra (1994). This implies that the implicit estimates for $\beta$ by Feenstra are even larger than the CD- and TD-OLS estimates for $\beta$ that are known to be biased upward. A possibility is that the variables were measured with significant errors. As a measurement error on the right-hand-side variable shrinks the OLS estimates towards zero, the upward bias of the TD- and CD-OLS estimates for $\beta$ would have been reduced by the measurement error, increasing the estimates for $\sigma$ in turn.

Feenstra (1994) noted potential measurement errors in the variables and hence he employed the GMM estimation method to overcome the problem. If Feenstra's estimates are asymptotically unbiased, the fact that the TD- and CD-OLS estimates for $\beta$ are smaller than his estimates will imply that the downward bias from the measurement error is so large that its effect is more than to just offset the upward bias caused by the truncation and censoring.

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6 The observations falling in the second and fourth quadrants in the x-y space have been truncated or censored.
7 A proof for this result is available from the author on request.
Although it was adopted to make the estimates comparable with the earlier results by Feenstra, the approach of pooling the data over time periods to estimate $\sigma$ has an important drawback as an exact-index-number approach to aggregation. The aggregation method will no longer be consistent over time. An index-number approach to aggregation would be consistent over time if it did not change the mechanical method of substituting observed values into the same known formula over time. When the estimation of an unknown constant involves a time series data (as it was the case with the pooled data), the consistency will be lost because when new observations become available the constant will have to be re-estimated.

To see how the new index formula performs when the consistency feature of an index-number formula is maintained, the elasticity of substitution for each pair of periods, $t-1$ and $t$, is computed only using the cross-sectional observations in $I(t)$. An average measure of the elasticity of substitution for each product obtained through this approach, with each $I(t)$ set censored, is reported in the second last column, labeled "IOLS" estimate. The reported estimates for $\sigma$ are computed by $(1/\bar{\beta}_3 + 1)$ where $\bar{\beta}_3$ is the average of the time series of the estimates for $\beta$ based on the individual CD-I(t)'s. The CD-IOLS estimates for $\sigma$ are similar to their CD-OLS counterparts, especially when $I(t)_1$ is used. For TV receivers and Typewriters, the estimates for two alternative intersection sets are quite different. In the case of TV receivers, the sensitivity to the change of the intersection sets appears to be partly due to the small numbers of observations in the $I(t)_2$ sets that are used to estimate $\beta$ in each period. The average number of observations in the $I(t)_2$ sets for TV receivers is 3.8, while those for the other products except Silver Bullion range from 7.5 (Stainless Steel Bars) to 11.4 (Cotton Knit Shirts). Silver Bullion has a relatively low average number of observations of 5.1, but the estimates are very stable over the two alternative intersection sets. This is not surprising because Silver Bullion is a highly homogeneous product, especially when produced within the same exporter country. Thus, the price and quantity measures that are aggregates of all varieties within each country must be relatively free of errors for Silver Bullion.

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8 For all the products and time periods, there were a few cases where the CD-IOLS estimates for $\beta$ were zero (when the order of the censored $I(t)$ was less than 2) or negative with highly insignificant t ratios. Those estimates were
Table 2: OLS Estimates of $\alpha$ and $\beta$ ($= 1/(\sigma - 1)$) using the pooled CD data set

<table>
<thead>
<tr>
<th>Products</th>
<th>Intersection Set = $I(t)_1$</th>
<th>Intersection Set = $I(t)_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Shoes (413)</td>
<td>TV (127)</td>
</tr>
<tr>
<td>Intercept ($\alpha$)</td>
<td>0.0017 (0.0092)*</td>
<td>0.0184 (0.0263)</td>
</tr>
<tr>
<td></td>
<td>Shirts (634)</td>
<td>Typewr. (300)</td>
</tr>
<tr>
<td></td>
<td>0.0140 (0.0097)</td>
<td>$-$0.0099 (0.0177)</td>
</tr>
<tr>
<td>Slope ($\beta$)</td>
<td>0.0769 (0.0101)</td>
<td>0.1093 (0.0207)</td>
</tr>
<tr>
<td></td>
<td>S Bars (216)</td>
<td>Gold (236)</td>
</tr>
<tr>
<td></td>
<td>0.1055 (0.0105)</td>
<td>0.1154 (0.0175)</td>
</tr>
<tr>
<td></td>
<td>S Sheets (345)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0845 (0.0149)</td>
<td>0.0281 (0.0038)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0021 (0.0042)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Products</th>
<th>Intersection Set = $I(t)_1$</th>
<th>Intersection Set = $I(t)_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Shoes (213)</td>
<td>TV (58)</td>
</tr>
<tr>
<td>Intercept ($\alpha$)</td>
<td>0.0035 (0.0075)</td>
<td>$-$0.0095 (0.0451)</td>
</tr>
<tr>
<td></td>
<td>Shirts (247)</td>
<td>Typewr. (189)</td>
</tr>
<tr>
<td></td>
<td>0.0175 (0.0158)</td>
<td>$-$0.0174 (0.0208)</td>
</tr>
<tr>
<td>Slope ($\beta$)</td>
<td>0.0639 (0.0106)</td>
<td>0.1217 (0.0396)</td>
</tr>
<tr>
<td></td>
<td>S Bars (160)</td>
<td>Gold (83)</td>
</tr>
<tr>
<td></td>
<td>0.1303 (0.0161)</td>
<td>0.0760 (0.0253)</td>
</tr>
<tr>
<td></td>
<td>S Sheets (245)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.1328 (0.0209)</td>
<td>0.0343 (0.0070)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0170 (0.0043)</td>
</tr>
</tbody>
</table>

*: The number of observations.
+: Standard errors in parentheses.

Table 2 reports the pooled CD-OLS estimates for $\alpha$ and $\beta$, together with their standard errors. The upper panel shows the estimates when the intersection set is defined as $I(t)_1$ while the lower panel shows the estimates when $I(t)_2$ is used. All $\alpha$ coefficients, except for Gold with $I(t)_1$ where $\alpha$ is just significant at 5%, are insignificantly different zero with high probability values. This result implies that the Vartia index is an unbiased measure of the average price changes within the intersection set. It also implies that the inclusion of an intercept term in the OLS formula does not have significant effect on the *exactness* feature of the new index formula when it is used below to compute price indices substituting the OLS estimates for $\beta$.

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excluded in computing the average of the estimates for $\beta$. Furthermore, those estimates are replaced with a zero when the time series of the price indices are constructed below.
Figure 1: Athletic Shoes

Figure 2: Cotton Knit Shirts

Figure 3: Stainless Steel Bars
Figure 4: Carbon Steel Sheets

Figure 5: Color TV Receivers

Figure 6: Portable Typewriters
Figures 1–8 show time-series plots of the Vartia, Feenstra, and the new indices using the pooled CD-OLS and cross-sectional CD-IOLS estimates for $\beta$ for each of the eight products. Only the indices based on $I(t)_2$ are reported since those based on $I(t)_1$ are very close to Feenstra’s index ($\pi_1$ in the figures of Feenstra [1994]). The plots show that the new indices based on both CD-OLS and CD-IOLS estimates trace closely the movement of Feenstra's index. In almost all cases, the
direction of change in the new index series matches with the direction of change in the Feenstra's series.\textsuperscript{9}

In general, the new indices based on the CD-IOLS estimates are closer to the series observed by Feenstra than the new indices based on the CD-OLS estimates are. The only exception is the indices for TV Receivers where a significant difference between the estimate for $\sigma$ based on $I(t)_1$ and the estimate based on $I(t)_2$, due to small sample sizes, has been noticed. As one would expect for a highly homogeneous product, the four series are almost undistinguishable for Gold Bullion and Silver Bullion.

As discussed in the previous section, the new price index at each period can be decomposed into three parts – the change in the aggregate price of the varieties in the intersection set $I$, the change due to the introduction of "new" varieties ($I^{\text{new}}$), and the change due to the disappearance of "old" varieties ($I^{\text{old}}$). Figure 9a shows a time-series plot of the three decomposed indices together with the overall price index for Cotton Knit Shirts.\textsuperscript{10} Note that the series are the two-period indices at each point of time and not "chained" indices. Naturally, the indices for the effect of disappeared varieties are higher than unity, implying that the effect increases the overall price indices, while those for the effect of new varieties are lower than unity, implying that the effect decreases the overall price indices. Thus, in a given time period, the new index will be lower than the Vartia index if the effect of having new varieties dominates the effect of losing old varieties, and it will be higher than the Vartia index if the opposite is true. The Figure shows that in most years, except the years around 1980, the effect of having new varieties is dominant over the effect of disappeared varieties in determining the overall price level of imported shirts.

\textsuperscript{9} The only exceptions are for the indices for Shoes in 1981 and for Shirts in 1986 in the case of the pooled estimation, and for Shirts in 1975 and 1982 and for TV Receivers in 1976 in the case of individual cross-sectional estimation.

\textsuperscript{10} Plots for the other products are not included to save space. Cotton Knit Shirts is chosen since it shows the most conspicuous changes in the series.
Figure 9a: Effect of New & Old Varieties on Price
Cotton Knit Shirts - pooled CD-OLS, I(t)2

Index of Price Change

Old
Vartia
New
Overall

Year
65 67 75 77 79 81 83 85 87

Figure 9b: Numbers of New & Old Varieties
Cotton Knit Shirts - I(t)2

Numbers of Varieties (countries)

Old  I  New

Year
65 67 69 71 73 75 77 79 81 83 85 87

-40 -30 -20 -10 0 10 20 30 40 50
The most conspicuous feature of the plot is that the effects of both new varieties and old varieties increase over time. Although not shown here, the phenomenon is true for all the other products apart from Gold and Silver for which the effects of new and old varieties are negligible. Figure 9b provides an explanation for this phenomenon. It shows that the number of varieties in each of $I_{\text{new}}$ and $I_{\text{old}}$ increases significantly over the sample period, while the number of varieties in $I(t)_2$ is stagnant after an abrupt fall in 1974. This, together with the increasing effects of new and old varieties, implies that the group of industrialized countries that had been regularly exporting Cotton Knit Shirts to the U.S. had been losing its market share to new exporter countries – mostly developing countries. Note, however, that the increasing numbers of new varieties and old varieties do not necessarily mean that a large number of countries are an irregular exporter. This is the case because $I(t)_2$ excludes developing countries. The increasing sizes of $I_{\text{new}}$ and $I_{\text{old}}$ are mainly due to the increasing number of developing countries that are mostly a regular exporter.

4. Concluding Remarks

The new index introduced in this paper is exact for the CES preference ordering even when there are new and/or disappeared varieties. The substitution of $\sigma$ with a known function of available prices and quantities has been facilitated by utilizing the CES restriction that the elasticity of substitution between the varieties in the set $I$ must be the same as the elasticity of substitution between the varieties in the whole choice set. As it can be computed by simply substituting observable quantities and prices into a known formula, it will no doubt simplify many analyses that require an aggregate over new and/or disappeared varieties as well as existing varieties. The new index would also prove useful for the analyses that require measurements of the separate effects of new, disappeared and existing varieties on the aggregate price index.

Although exact theoretically, the variables in the new index formula may include noises and measurement errors in practice, raising the question of statistical consistency of the formula. As observed in the previous section, when the new index formula was applied to the data analysed by Feenstra, such measurement errors could result in a bias in the new index. To benefit from the advantages of the simpleness and methodological consistency that an exact index number formula provides, it will be important to use values that are largely free of such errors.
References


