

# A unified model of international trade with increasing returns and oligopoly

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## Abstract

A two-country oligopolistic model with increasing returns is developed to make clear the trade patterns and gains from trade allowing for the possibility of complete specialization in a country. The two countries differ only in size measured by factor endowments. It is shown that which country exports the increasing-returns good and benefits from trade depends on the relative difference between the two parameters one of which indicates the magnitude of scale economies and the other of which the number of oligopolistic firms. Two seminal outcomes by Markusen (1981) and Ethier (1982) prove to arise as a polar case by choosing the two parameters properly.

*Keywords:* Increasing returns; Oligopoly; Trade patterns; Gains from trade

*JEL Classification:* F10; F12

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# 1 Introduction

The past twenty-five years have witnessed much literature that explores the positive and normative implications of increasing returns and imperfect competition for international trade.<sup>1</sup> As is well-known, focusing on external economies of scale which are compatible with perfect competition, the first generation, such as Kemp and Negishi (1970) find that a country gains from trade if its production of the increasing-returns good increases after the opening of trade.<sup>2</sup> Then, incorporating external economies of scale into a two-country Ricardian model in which the countries differ only in their labor endowment, Ethier (1982) conclude that the large country exports the increasing-returns good, which results in its gains from trade, whereas the small country may lose from trade.

Markusen (1981), on the other hand, gives a first formulation of a two-country model of international oligopoly to explore trade patterns, factor price equalization, and gains from trade. One of his main results is that a small country necessarily gains from trade, while the large country possibly loses from trade when constant returns prevail in all sectors. This result is worth noting since it is exactly the opposite to Ethier's (1982). Based on this finding, he also shows that the production expansion condition continues to constitute a sufficient condition for gainful trade in a context of oligopoly; if a country's output of the non-competitive good under free trade exceeds that under autarky, it gains from trade.

Then, what conclusion will be established regarding trade patterns and gains from trade under increasing returns and imperfect competition altogether? In view of the con-

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<sup>1</sup>A comprehensive treatment is provided by Helpman (1984), Helpman and Krugman (1985) and Wong (1995).

<sup>2</sup>Markusen and Melvin (1984) call it the production expansion condition.

clusions in Ethier (1982) and Markusen (1981), it is fair to say that the exporting country of the non-competitive and increasing-returns good necessarily gains from trade. Indeed, this guess has already been confirmed by Markusen and Melvin (1984) and Schweinberger (1996) in a general model. However, their conditions are imposed on *endogenous* variables. To our knowledge, there is no work that attributes the gains from trade to *exogenous* variables such as factor endowments.

And Markusen (1981) shows that “many, but not all, of the results derived in the previous sections continue to hold with increasing returns”. Such validities with increasing returns are based on the assumption that the interior Cournot-Nash equilibrium is realized, i.e., both countries diversify. However, increasing returns make the possibility of complete specialization more likely.<sup>3</sup> Therefore, it remains an open question what result we can obtain about trade patterns and gains from trade by allowing for such a possibility. This paper constructs an analytical framework which addresses the above questions. A virtue we should stress is that two seminal results by Ethier (1982) and Markusen (1981) will arise as polar cases. In this sense, our model provides a simple reconciliation between the two works.

We adopt a two-country Ricardian model containing two key parameters one of which measures the degree of scale economies and the other of which measures that of monopoly power.<sup>4</sup> If the former is sufficiently large such that the effect of increasing returns dom-

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<sup>3</sup>Someone may criticize that Markusen’s (1981) assumption of incomplete specialization is not problematic since he employed a Heckscher-Ohlin technology. But, even in Heckscher-Ohlin settings, the convex segment of the production possibility frontier dominates, which can be approximated by a Ricardian setting when increasing returns are sufficiently strong.

<sup>4</sup>While Ruffin (2003a, 2003b) also developed a two-country oligopolistic Ricardian model, his attention was focused on the validity of the law of comparative advantage and the assumption of constant returns is retained throughout the paper.

inates, the large country exports the increasing-returns good; Ethier's (1982) conclusion follows. On the contrary, Markusen's (1981) result that the small country exports the increasing-returns good survives if the latter is large enough, i.e., each oligopolist's market power is so strong. We will derive the critical value between them which determines the exporter of the increasing-returns good and the gainer country.

The paper proceeds as follows. Section 2 builds up a basic model and describes the autarkic equilibrium. Extending it to a two-country world, Section 3 characterizes a trading equilibrium and derives the propositions concerning trade patterns. Section 4 turns to the issue of gains from trade. Section 5 sums up the conclusion of the paper. Two appendices deal with technical issues in the main text.

## 2 An autarkic equilibrium

A two-country (home and foreign), two-good (goods 1 and 2), one-factor (labor) model is developed. This section describes the autarkic equilibrium of a country, say, the home country. Exactly the same argument applies to the foreign country.

Let us begin with specifying the production technology. Good 2 is produced under constant returns and perfect competition, which serves as a numeraire. By choosing units properly, one unit of labor produces one unit of good 2 (numeraire), which, together with the competitive condition, makes the wage rate equal to unity as long as good 2 is positively supplied. Sector 1 is oligopolized by  $n \geq 2$  identical firms and their technology is characterized by increasing returns:

$$x_i = l_i^\alpha, \quad \alpha > 1, \quad i = 1, \dots, n,$$

where  $x_i$  and  $l_i$  are a respectively representative firm's output and labor input in sector 1. Thus, the corresponding cost function is given by  $wx_i^{1/\alpha}$  with  $w$  denoting the wage rate.

The demand side is now formulated. The community utility function is homothetic and takes a Cobb-Douglas form:

$$u = C_1^\gamma C_2^{1-\gamma}, \quad \gamma \in (0, 1),$$

where  $u$  is the utility level and  $C_1$  and  $C_2$  are the consumption of each good. The demand function of good 1 associated with this preference is

$$C_1 = \frac{\gamma I}{p},$$

where  $p$  and  $I$  are the price of good 1 and the national income.

Based on these set-ups, the autarkic equilibrium is described. First of all, note that the autarkic equilibrium, if exists, must involve a positive supply of good 2 due to the Cobb-Douglas preference. A representative firm, say, firm  $i$ 's profit is then defined as  $px_i - x_i^{1/\alpha}$  from  $w = 1$ .

Following most of the existing literature, each oligopolistic firm is supposed to maximize profit by assuming that any oligopolist consumes only good 2.<sup>5</sup> Assuming that the oligopolistic firms play a Cournot-Nash game and that they take the national income as

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<sup>5</sup>When each oligopolist consumes both goods, the assumption of profit maximization is vulnerable as Kemp and Okawa (1995) and Kemp and Shimomura (1995, 2002) show. However, this problem can be overcome by assuming that oligopolists consume only the numeraire good.

given, a firm's first-order condition for profit maximization is given by<sup>6</sup>

$$p \left(1 - \frac{1}{n}\right) - \frac{1}{\alpha} x^{\frac{1}{\alpha}-1} = 0. \quad (1)$$

In the present general equilibrium model,  $p$  and  $I$  are determined through the following system of equations:

$$\frac{\gamma I}{p} = nx \quad (2)$$

$$I = pnx + L - nx^{\frac{1}{\alpha}}, \quad (3)$$

where (2) represents the autarkic market-clearing condition of good 1 and (3) the definition of the national income. Hence, solving for  $p$  gives

$$p = \frac{\gamma}{1-\gamma} \frac{L - nx^{\frac{1}{\alpha}}}{nx}.$$

Substituting this equilibrium price into (1), the first-order condition for an *interior* autarkic equilibrium is derived as

$$\frac{\gamma}{1-\gamma} \frac{L - nx^{\frac{1}{\alpha}}}{nx} \left(1 - \frac{1}{n}\right) - \frac{1}{\alpha} x^{\frac{1}{\alpha}-1} = 0, \quad (4)$$

whose solution is obtained by

$$x^A = \left\{ \frac{\alpha\gamma(n-1)L}{n[\alpha\gamma(n-1) + (1-\gamma)n]} \right\}^\alpha \equiv (l^A)^\alpha, \quad (5)$$

where the superscript  $A$  indicates the autarkic equilibrium. Thus, we can immediately state:

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<sup>6</sup>Taking account of the effect of each oligopolistic firm's output on the national income, Tawada and Okawa (1995) prove that the marginal revenue is smaller if monopolistic firms maximize profits by allowing for the effect of their output on the national income than taking it as given. Hence, the profit-maximizing solution with this income effect taken into account is less than that when the national income is given to each monopolistic firm.

**Lemma 1.** *There uniquely exists an autarkic equilibrium in each country if and only if*

$$\frac{\alpha}{\alpha - 1} \geq n \geq 2.$$

*Proof.* As mentioned, any autarkic equilibrium, if exists, must involve  $x^A > 0$  but this is not always the case since increasing returns can make oligopolistic firms' profit negative. In order to ensure the positivity of profits, substitute  $x^A$  into the definition of profits to get

$$\pi^A \equiv \frac{[n - \alpha(n - 1)]\gamma L}{n[\alpha\gamma(n - 1) + (1 - \gamma)n]},$$

which is the per-firm maximized profit. Hence, we easily see that  $\pi^A \geq 0$  if and only if

$$n \leq \frac{\alpha}{\alpha - 1}.$$

When the parameter set satisfies this inequality,  $x^A > 0$  is safely guaranteed. Moreover, since  $n$  is more than 2, the lemma is now proved.

Having proved the unique existence of autarkic equilibrium, let us address the stability quickly. Following the standard argument, the output is assumed to be adjusted according to the difference between marginal revenue and marginal cost. Letting respectively the first and second terms in the left-hand side in (4) denote  $MR(x, L)$  and  $MC(x)$ , the stability condition becomes

$$MR_x(x^A, L) - MC'(x^A) < 0,$$

where  $MR_x(\cdot)$  is the partial derivative of  $MR(\cdot)$  with respect to  $x$ . Utilizing the fact that  $x^A$  is the solution to (4), the above stability condition is equivalent to

$$-\frac{MR_x(x^A, L) x^A}{MR(x^A, L)} > -\frac{MC'(x^A) x^A}{MC(x^A)},$$

which reduces to

$$\frac{n(x^A)^{\frac{1}{\alpha}}}{\alpha [L - n(x^A)^{\frac{1}{\alpha}}]} > -\frac{1}{\alpha}.$$

This condition is necessarily satisfied since the left-hand side is positive, while the right-hand side is negative. Note that this condition constitutes not only the stability condition but the second-order condition which has to be met as another regularity condition. This result is summarized in:

**Lemma 2.** *The stability and the second-order conditions of autarkic equilibrium are satisfied for any set of parameters,  $\alpha$  and  $n$ .*

This section is closed by giving the utility level in the autarkic equilibrium. Since we have assumed a representative consumer, a country's welfare can be measured by its indirect utility function, which is given by

$$V(p, I) \equiv \gamma^\gamma (1 - \gamma)^{1-\gamma} \frac{I}{p^\gamma},$$

from the Cobb-Douglas preference. Substitution of (5) into the solutions of  $p$  and  $I$  in (2) and (3) and further substitution of them into  $V(p, I)$ , we have

$$\begin{aligned} V(p^A, I^A) &= \gamma^\gamma (1 - \gamma)^{1-\gamma} \left\{ \frac{\gamma L}{[\alpha\gamma(n-1) + (1-\gamma)n] \left\{ \frac{\alpha\gamma(n-1)L^*}{n[\alpha\gamma(n-1) + (1-\gamma)n]} \right\}^\alpha} \right\}^\gamma \\ &\quad \times \frac{nL}{\alpha\gamma(n-1) + (1-\gamma)n}. \end{aligned} \tag{6}$$

### 3 Free trade equilibria and trade patterns

#### 3.1 Characterization of free trade equilibria

This and subsequent sections extend the above model to a two-country world consisting of the home and foreign countries whose labor endowment is possibly different. All of the foreign variables are distinguished by attaching an asterisk (\*). Due to the identical preference, technology, and number of oligopolistic firms between the countries and symmetries among oligopolistic firms, the first-order condition for profit maximization of a representative firm in each country is respectively given by

$$p \left[ 1 - \frac{x}{n(x + x^*)} \right] - \frac{1}{\alpha} x^{\frac{1}{\alpha}-1} = 0 \quad (7)$$

$$p \left[ 1 - \frac{x^*}{n(x + x^*)} \right] - \frac{1}{\alpha} x^{*\frac{1}{\alpha}-1} = 0. \quad (8)$$

In a similar manner to the autarkic equilibrium, the market-clearing price and national income in the countries are determined through the system of equations:

$$\frac{\gamma(I + I^*)}{p} = n(x + x^*) \quad (9)$$

$$I = pnx + L - nx^{\frac{1}{\alpha}} \quad (10)$$

$$I^* = pnx^* + L^* - nx^{*\frac{1}{\alpha}}, \quad (11)$$

which yields

$$p = \frac{\gamma}{1 - \gamma} \frac{L + L^* - n \left( x^{\frac{1}{\alpha}} + x^{*\frac{1}{\alpha}} \right)}{n(x + x^*)}.$$

Thus, substituting this market-clearing price into (7) and (8) yields

$$\frac{\gamma}{1 - \gamma} \frac{L + L^* - n \left( x^{\frac{1}{\alpha}} + x^{*\frac{1}{\alpha}} \right)}{n(x + x^*)} \left[ 1 - \frac{x}{n(x + x^*)} \right] - \frac{1}{\alpha} x^{\frac{1}{\alpha}-1} = 0 \quad (12)$$

$$\frac{\gamma}{1-\gamma} \frac{L+L^* - n\left(x^{\frac{1}{\alpha}} + x^{*\frac{1}{\alpha}}\right)}{n(x+x^*)} \left[1 - \frac{x^*}{n(x+x^*)}\right] - \frac{1}{\alpha} x^{*\frac{1}{\alpha}-1} = 0. \quad (13)$$

In the subsequent arguments, we shall intensively use the optimality conditions in terms of labor inputs in order to apply Ethier's (1982) geometry in the  $l - l^*$  space. Then, the system of (12) and (13) is rewritten as

$$\frac{\gamma}{1-\gamma} \frac{L+L^* - n(l+l^*)}{n(l^\alpha + l^{*\alpha})} \left[1 - \frac{l^\alpha}{n(l^\alpha + l^{*\alpha})}\right] - \frac{1}{\alpha} l^{1-\alpha} = 0 \quad (14)$$

$$\frac{\gamma}{1-\gamma} \frac{L+L^* - n(l+l^*)}{n(l^\alpha + l^{*\alpha})} \left[1 - \frac{l^{*\alpha}}{n(l^\alpha + l^{*\alpha})}\right] - \frac{1}{\alpha} l^{*1-\alpha} = 0 \quad (15)$$

Figures 1-3 give three candidates for the trading equilibrium. In the figures, the mountain-shaped locus  $ONB$  is the home firm's reaction curve, i.e., the input pair on  $ONB$  satisfies (14). Similarly, the locus  $ONB'$  is the foreign firm's reaction curve such that (15) is met.<sup>7</sup>

We see, from (14) and (15), at least one Cournot-Nash equilibrium involves an symmetry, i.e.,  $l^N = l^{*N}$  where the superscript  $N$  stands for the interior Nash equilibrium. In such a symmetric Nash equilibrium given by  $N$  in the figures, each firm's optimal input becomes

$$l^N = l^{*N} = \frac{\alpha\gamma(2n-1)(L+L^*)}{2n[\alpha\gamma(2n-1) + 2(1-\gamma)n]}. \quad (16)$$

However, this is not the only equilibrium and multiple equilibria are possible depending on the parameter sets. In what follows, how the three possibilities arise is analyzed.

First of all, let us divide the three figures into Figure 1 and Figures 2 and 3. The home firm's reaction curve is flatter than the foreign firm's at  $N$  in Figure 1, while the opposite holds in Figures 2 and 3. As shown in Appendix A, Figure 1 emerges if and

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<sup>7</sup>Appendix A examines a few properties of the reaction curve.

only if

$$n > \frac{2\alpha - 1}{2(\alpha - 1)}, \quad (17)$$

which requires that the home firm's reaction curve be flatter than the foreign firm's at  $N$ . On the other hand, the condition under which the other two are obtained is given by

$$n < \frac{2\alpha - 1}{2(\alpha - 1)}. \quad (18)$$

Conditions (17) and (18) have a clear interpretation. Figure 1 is basically the same as Ethier's (1982) externality model, while Figures 2 and 3 correspond to Markusen's (1981) oligopolistic model. Condition (17) states that the equilibrium in our model is more likely to resemble Ethier's (1982) as the degree of increasing returns ( $\alpha$ ) is sufficiently large.<sup>8</sup> On the contrary, when the number of firms ( $n$ ) is so small that (18) holds, i.e., the effect of imperfect competition is far more dominant, the model behavior is more likely to approach Markusen's (1981).

In the figures, the arrows indicate the dynamic process of the world economy. They are based on the following adjustment process which is standard in oligopoly theory:

$$\dot{l} = \widetilde{MR}(l, l^*, L + L^*) - MC(l) \quad (19)$$

$$\dot{l}^* = \widetilde{MR}(l^*, l, L + L^*) - MC(l^*), \quad (20)$$

where  $\widetilde{MR}(\cdot)$  denotes the first term and  $MC(\cdot)$  the second term in the left-hand side in (14) and (15).

We are now ready to consider how Figures 2 and 3 are sub-divided. Both figures are similar in the sense that the home firm's reaction curve is steeper than the foreign firm's

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<sup>8</sup>Note that the right-hand side in (17) and (18) is decreasing in  $\alpha$ .

at  $N$ .<sup>9</sup> The difference lies in that each firm's maximized profit obtained at  $N$  is negative in Figure 2, while it is positive in Figure 3. In other words, the interior Nash equilibrium is meaningful only in Figure 3 from economic viewpoints. Therefore,  $N$  in Figure 2 can not constitute the free trade equilibrium.<sup>10</sup>

To verify the above, let us substitute  $l^N = l^{*N}$  into the definition of each firm's profit. Then, the maximized profit at  $N$  is obtained as

$$\pi^N = \frac{\gamma[2(1-\alpha)n + \alpha](L + L^*)}{2n[\alpha\gamma(2n-1) + 2(1-\gamma)n]},$$

which turns out to be non-negative if and only if

$$n \leq \frac{\alpha}{2(\alpha-1)}. \quad (21)$$

That is, in order to assure the interior Nash equilibrium such that  $l^N = l^{*N}$ , an additional condition given by (21) must be imposed. Note that (21) implies (18). In sum, we have arrived at that Figure 1 is drawn under (17), that Figure 2 is drawn under  $(2\alpha-1)/[2(\alpha-1)] > n > \alpha/2[(\alpha-1)]$ , and that Figure 3 is drawn under (21).

### 3.2 Patterns of specialization and trade

Resorting to Figures 1-3, we can derive the propositions on patterns of specialization and trade. See Figure 2 in which the locus  $SS$  gives a saddle path associated with the steady state  $R$  and the line  $OT$  goes through the intersection of  $BB'$  and  $SS$ . Letting

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<sup>9</sup>In Figures 2 and 3, there are three intersections of the reaction curves. However,  $N$  is the only interior equilibrium because the others are on the upward-sloping segment of a firm's reaction curve, which violates the second-order condition for profit maximization. Moreover, even if both reaction curves are downward-sloping on these points, such points constitute a saddle point. Hence, any path except for the saddle path will diverge and approach one of  $B'$ ,  $N$ , and  $B$ , which enables us to put the side intersections out of consideration.

<sup>10</sup>Note that the same is true of Figure 1 in which Ethier's type equilibrium is described.

$\psi$  be the slope of  $OT$  in the figure, we now find out the following results on patterns of specialization and trade depending on the parameters:

**Proposition 1.** *Suppose  $L^* > L$  and that the dynamic behavior of the world economy follows the adjustment process defined in (19) and (20). Then, the following trade patterns are observed.*

(i) *The case under (17) in Figure 1: the foreign country diversifies and exports good 1, while the home country specializes in good 2 and imports good 1;*

(ii) *The case under (18) in Figures 2 and 3: if the relative size of the two countries satisfies  $L^*/L > \psi$ , the foreign country diversifies and exports good 1, while the home country specializes in good 2 and imports good 1. On the other hand, if (21) and  $L^*/L < \psi$  hold, both countries diversify and produce the same amount of good 1. Then, the home country exports good 1.*

*Proof.* First of all, two points must be noted. First, any autarkic equilibrium is captured by the intersection of  $BB'$  and the straight line connecting the origin and the endowment point on  $WW'$ .<sup>11</sup> Second, the reaction curve outside the rectangle shaped by the origin and the endowment point on  $WW'$  can be out of consideration.<sup>12</sup> Having this in mind, let us prove each case separately.

In part (i) which is associated with Figure 1, starting from the autarkic equilibrium,

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<sup>11</sup> $WW'$  gives the per-firm labor endowment pair and formally given by  $l^* = (L + L^*)/n - l$ .

<sup>12</sup>For the details, see Ethier (1982) and Suga (2005).

say  $K$ , the economy converges to  $B'$  at which the foreign country diversifies and the home country specializes in good 2. This pattern holds for any  $L^* > L$ . The trade pattern is trivially obtained from the specialization pattern.

The former half of part (ii) is proved as follows. Under  $L^*/L > \psi$ , the economy goes to  $B'$ . Thus, the rest of the argument follows that in part (i).<sup>13</sup>

On the other hand, in the latter half of part (ii) such that  $L^*/L < \psi$  holds, the world converges to  $N$  from  $I$ . At  $N$ ,  $l^N = l^{*N} > 0$  occurs due to the positivity of the equilibrium profits and the home country exports good 1 since its demand of good 1 is less than the foreign one.

Proposition 1 asserts that the patterns of specialization and trade obtained in Ethier's (1982) externality model and Markusen's (1981) oligopolistic model arise as a special case in our model. According to part (i) in the proposition such that the number of oligopolistic firms is sufficiently large, the free trade equilibrium becomes similar to Ethier's (1982) in which the large country exports the increasing returns good. In contrast, Markusen's (1981) trade pattern proposition, i.e., the small country exports the non-competitive good, is observed if the number of oligopolistic firms is small enough to satisfy (21).<sup>14</sup>

The intermediate case given in the former half of part (ii) and Figure 2 is a new finding. In this case, one may guess that the trade pattern resembles Markusen's (1981) diversifying one since Figures 2 and 3 are the same in the sense that the home firm's reaction curve is steeper than the foreign firm's at  $N$ . However, Proposition 1 clarifies that such a guess is incorrect and that the pattern of specialization and trade becomes like

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<sup>13</sup>Note that any path converging to  $N$  is not realized since each firm's profit evaluated there is negative.

<sup>14</sup>Strictly speaking,  $L^*/L < \psi$  must also be met.

Ethier (1982). That is, the equilibrium involves complete specialization to the numeraire good by the small country.

## 4 Gains and losses from trade

It is well-known that trade patterns do not affect gains from trade in the Arrow-Debreu framework. However, in imperfectly competitive models, particularly, under oligopoly without free entry, the direction of trade crucially affects the gains from trade.<sup>15</sup> The same is true of our model as well. The following lemma gives a helpful preliminary in discussing gains from trade:

**Lemma 3.** *Suppose that any active oligopolistic firm's equilibrium profit is non-negative. Then, free trade is beneficial to the country whose production of good 1 expands after the opening of trade.*

*Proof.* Attaching a subscript  $T$  to denote the variable at the trading equilibrium and letting  $E(p, u)$  be the expenditure function, we have the following inequalities.

$$\begin{aligned}
 E(p^T, u^T) &= p^T C_1^T + C_2^T \\
 &= p^T X_1^T + X_2^T \\
 &= n \left[ p^T (l^T)^{\alpha-1} - 1 \right] l^T + L \\
 &> n \left[ p^T (l^A)^{\alpha-1} - 1 \right] l^A + L \\
 &= np^T (l^A)^\alpha + L - nl^A
 \end{aligned} \tag{22}$$

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<sup>15</sup>See Markusen and Melvin (1984) and Schweinberger (1996).

$$\begin{aligned}
&= p^T X_1^A + X_2^A \\
&= p^T C_1^A + C_2^A \\
&> E(p^T, u^A),
\end{aligned} \tag{23}$$

where  $X \equiv nx$ , (22) follows from the fact that  $(pl^{\alpha-1} - 1)l$  is increasing in  $l$  given  $p$ , and (23) is implied by expenditure minimization. Therefore,  $l^T > l^A$  or  $x^T > x^A$  is sufficient for  $u^T > u^A$ .

The above lemma immediately enables us to show that each country gains from trade in a special case with  $L = L^*$ :

**Lemma 4.** *Both countries gain from potential trade under  $L = L^*$  and*

$$\frac{\alpha}{2(\alpha - 1)} \geq n \geq 2.$$

*Proof.* The lemma is easy to prove by making use of Figure 3. In the figure,  $A$  and  $N$  respectively constitute the autarkic and trade equilibrium. Starting from  $A$ , the world economy converges to  $N$  on the stable path. Accordingly, each country's output of good 1 increases after starting trade. Pulling this fact with Lemma 3, we immediately arrive at Lemma 4.

Lemma 4 is alternatively described in Figure 4. In the figure, the bold locus which is strictly convex is a country's production possibility frontier. Lemma 4 states that the equilibrium moves from  $A$  to  $F$ , which results in the country's gains from trade. The

intuition behind it is basically the same as Markusen's (1981) case; the opportunity to trade makes each oligopolistic firm behave more competitively than under autarky, which enhances the country's welfare. Moreover, the presence of increasing returns, together with the output expansion, lowers each firm's average cost and increases its profit, which is another force for positive gains from trade.

Combining Lemma 3 with Proposition 1, we can safely say that the exporter of good 1 necessarily gains from trade stated in:

**Proposition 2.** *If the conditions for Figures 1 and 2 to be drawn are satisfied, the foreign country gains from trade, whereas the home country gains from trade if the parameter set yields Figure 3 and  $L^*/L < \psi$  holds.*

In view of Proposition 2, the rest of our main task is to examine whether the importer of good 1 gains from trade. Of course, the answer is conditional. The following proposition summarizes the results on the possibility of gainful trade of the importer of good 1:

**Proposition 3.** *The necessary and sufficient condition for the importer of good 1 to gain from trade is given as follows.*

*Figures 1 and 2:*

$$\frac{L^*}{L} > \left[ \frac{n}{\alpha\gamma(n-1) + (1-\gamma)n} \right]^{\frac{1}{\gamma(\alpha-1)}} - 1; \quad (24)$$

Figure 3:

$$\frac{L^*}{L} < \psi,$$

and

$$\begin{aligned} & \left\{ \left[ \frac{\alpha\gamma(n-1) + (1-\gamma)n}{\alpha\gamma(2n-1) + 2(1-\gamma)n} \right]^{\alpha-1} \left[ \frac{2n-1}{2(n-1)} \right]^\alpha \left( \frac{L+L^*}{L^*} \right)^{\alpha-1} \right\}^\gamma \\ & \times \frac{[\alpha\gamma(n-1) + (1-\gamma)n] \left\{ 2[\alpha\gamma(2n-1) + 2(1-\gamma)n] + \gamma[2(1-\alpha)n + \alpha] \frac{L+L^*}{L^*} \right\}}{2n[\alpha\gamma(2n-1) + 2(1-\gamma)n]} > 1. \end{aligned} \quad (25)$$

*Proof.* Since the proofs are nothing but tedious calculation exercises, only the outline of proof is sketched. Let  $p^T$  and  $I^T$  be the world price and national income of the importer of good 1 in the trading equilibrium. Then, substituting them into the indirect utility function  $V(p, I) \equiv \gamma^\gamma(1-\gamma)^{1-\gamma}I/p^\gamma$  and taking its ratio to  $V(p^A, I^A)$  yield

$$\frac{V(p^T, I^T)}{V(p^A, I^A)} = \left( \frac{p^A}{p^T} \right)^\gamma \frac{I^T}{I^A}.$$

All we have to do is substitute  $l$  and  $l^*$  in each trading equilibrium into the above index of welfare comparison and derive the condition for  $V(p^T, I^T)/V(p^A, I^A) > 1$ . In Figures 1 and 2,  $l^T = 0$  and

$$l^{*T} = \frac{\alpha\gamma(n-1)(L+L^*)}{n[\alpha\gamma(n-1) + (1-\gamma)n]},$$

are obtained.

On the other hand, Figure 3 gives

$$l^T = l^{*T} = \frac{\alpha\gamma(2n-1)(L+L^*)}{2n[\alpha\gamma(2n-1) + 2(1-\gamma)n]}.$$

Substitution of these into  $p$  and  $I$  or  $I^*$  and further substitution into the above index give us  $V(p^T, I^T)/V(p^A, I^A)$  of the importing country. After that, some calculations and rearrangements yield the necessary and sufficient conditions for the importer to gain from trade given by (24) and (25).

While Proposition 3 gives the necessary and sufficient conditions for the importer to gain from trade, it is of some use in considering the possibility of losses from trade. In Figures 1 and 2, the foreign country diversifies, which requires that the per-firm foreign labor endowment, i.e.,  $L^*/n$ , be larger than the vertical interception of the foreign firm's reaction curve given by

$$\frac{\alpha\gamma(L + L^*)}{n[\alpha\gamma(n - 1) + (1 - \gamma)n]}.$$

This condition is explicitly given by

$$\frac{L^*}{L} > \frac{\alpha\gamma(n - 1)}{(1 - \gamma)n}.$$

Pulling (24) together with the above 'diversification' condition, the home country loses if and only if

$$\left[ \frac{n}{\alpha\gamma(n - 1) + (1 - \gamma)n} \right]^{\frac{1}{\gamma(\alpha-1)}} - 1 > \frac{L^*}{L} > \max \left\{ 1, \frac{\alpha\gamma(n - 1)}{(1 - \gamma)n} \right\}. \quad (26)$$

(26) asserts that when both countries are sufficiently close to each other measured by labor endowments, the tendency for the small country to lose from trade is strengthened. This result is a confirmation of Ethier's (1982) Proposition 10 (p. 1261) concerning the possibility of trading losses for the small country.<sup>16</sup>

<sup>16</sup>Indeed, Ethier (1982) shows that the small country as well as the large country gains from trade when the large country incompletely specializes.

On the other hand, condition (25) requires the opposite in the case where the large country becomes the importer of good 1. Since it is almost trivial that the right-hand side in (25) is monotonically decreasing in  $L^*/L$ , (25) becomes more likely to be violated as  $L^*$  deviates from  $L$ , that is, the two countries become more asymmetric. If  $L^* - L$  is quite large, the foreign country loses from trade which is implicitly considered in Markusen (1981).

In sum, Proposition 3 provides a simple reconciliation of Ethier (1982) and Markusen (1981). If the number of firms is so large that Ethier's (1982) pattern of specialization and trade occurs, the large country necessarily gains, whereas the small country possibly loses from trade. On the contrary, if the number of firms is small enough to obtain Markusen's (1981) pattern of incompletely specialized trade, the small country always gains from trade, while the large country has a possibility of trading losses. In other words, which country gains or loses highly depends on the interaction between the scale economies and market power.

## 5 Concluding remarks

It has been thought difficult to deal with both increasing returns and imperfect competition in a unified framework of general equilibrium. Of course, there are two exceptions in the literature. The first strand assumes monopolistic competition like Dixit and Norman (1980) and Helpman and Krugman (1985), while the second allows for free entry in an oligopolistic model, e.g., Lahiri and Ono (1995) and Shimomura (1998). Due to the assumption of free entry and zero profit, the structure in these models turns to be similar to that in the neoclassical model, which makes the analysis simple and tractable.

On the other hand, there are few studies that formulate international oligopoly with restricted entry and increasing returns in a general equilibrium setting. As a recent contribution, Fujiwara and Shimomura (2005) construct an extended version of Markusen's (1981) model to show the validity of the factor proportions theory by allowing for arbitrary differences in the factor endowment ratio.

This paper has provided an alternative theory of international trade under increasing returns and international oligopoly. We believe that our model has two virtues. First, the results on trade patterns and gains from trade in Markusen (1981) and Ethier (1982) arise as a polar case by choosing parameters properly. Second, the optimal outputs in the autarkic and free trade equilibria are explicitly solved, which enhances the tractability of the model. Our geometric approach, which basically owes to Ethier (1979, 1982), has many potential directions of future researches. Among others, one of our next tasks is to reconcile our model with the two-factor setting in Fujiwara and Shimomura (2005) by relating the difference in factor abundance to gains from trade. Another direction of extension is to incorporate international economies of scale first formulated by Ethier (1979) and sophisticated by Suga (2005). In view of that international economies of scale induce a drastic change in implications relative to national economies of scale, such a direction is of another great importance.

## Appendix A: some properties of the reaction curve

This appendix examines some properties of the home firm's reaction curve. Invoke (14):

$$\begin{aligned} & \frac{\gamma}{1-\gamma} \frac{L+L^*-n(l+l^*)}{n(l^\alpha+l^{*\alpha})} \left[ 1 - \frac{l^\alpha}{n(l^\alpha+l^{*\alpha})} \right] - \frac{1}{\alpha} l^{1-\alpha} \\ \equiv & \widetilde{MR}(l, l^*, L+L^*) - MC(l) = 0. \end{aligned}$$

Let us begin with setting  $l^* = 0$  and computing the best response to it. Substituting  $l^* = 0$  and solving for  $l$ , we have

$$l = \frac{\alpha\gamma(n-1)(L+L^*)}{n[\alpha\gamma(n-1) + (1-\gamma)n]} > 0.$$

On the other hand, we see that the other interception is given by  $l^* = 0$  by setting  $l = 0$ , i.e., the home firm's reaction curve goes through the origin.

We next look at the slope of the reaction curve. Differentiating the above first-order condition with respect to  $l$  and  $l^*$  and rearranging, the absolute value of the slope of the reaction curve takes the form of

$$-\frac{dl^*}{dl} = \frac{\Gamma(l, l^*)}{\Delta(l, l^*)}, \quad (27)$$

where

$$\begin{aligned} \Gamma(l, l^*) & \equiv \widetilde{MR}_l(l, l^*, L+L^*) - MC'(l) \\ & = \frac{\gamma}{1-\gamma} \frac{1}{[n(l^\alpha+l^{*\alpha})]^3} \left\{ -n^2(l^\alpha+l^{*\alpha})[(n-1)l^\alpha+nl^{*\alpha}] \right. \\ & \quad \left. -nal^{\alpha-1}[L+L^*-n(l+l^*)][(n-1)l^\alpha+(n+1)l^{*\alpha}] - \frac{1-\alpha}{\alpha} l^{-\alpha} \right\} \\ \Delta(l, l^*) & \equiv \widetilde{MR}_{l^*}(l, l^*, L+L^*) \\ & = \frac{\gamma}{1-\gamma} \frac{1}{[n(l^\alpha+l^{*\alpha})]^3} \left\{ -n^2(l^\alpha+l^{*\alpha})[(n-1)l^\alpha+nl^{*\alpha}] \right. \\ & \quad \left. -nal^{\alpha-1}[L+L^*-n(l+l^*)][(n-2)l^\alpha+nl^{*\alpha}] \right\}, \end{aligned}$$

where the subscripts denote the partial derivative of  $\widetilde{MR}(\cdot)$  with respect to  $l$  and  $l^*$ . Note that  $\Delta(\cdot)$  is negative for any  $l$  or  $l^*$ , and parameters.

Considering the system of equations (14) and (15), the two reaction curves intersect on the 45° line. And there are two candidates for how they intersect. One possibility is that the home firm's curve cuts the foreign firm's from the above, i.e., the home firm's reaction curve is steeper than the foreign firm's. Another is the opposite to the first case, that is, the home firm's reaction curve is less steeper than the foreign firm's. Of course, this difference comes from the interaction of parameters. In particular, the relative magnitude of  $n$  and  $\alpha$  plays a significant role, which we will show. This difference will also make a big difference for patterns of specialization and hence gains from trade.

Evaluating (27) at  $l = l^*$ , it is simplified to

$$-\left. \frac{dl^*}{dl} \right|_{l=l^*} = \frac{\frac{-\gamma}{1-\gamma} \frac{(2n-1)l + \alpha(L+L^*-2nl)}{4nl^{\alpha+1}} + \frac{\alpha-1}{\alpha} \frac{1}{l^\alpha}}{\frac{-\gamma}{1-\gamma} \frac{n(2n-1)l + (n-1)\alpha(L+L^*-2nl)}{4n^2l^{\alpha+1}}}. \quad (28)$$

In the first case where the home firm's curve is steeper, (28) must exceed unity, which leads to

$$\frac{L + L^* - 2nl}{2nl} \frac{\alpha\gamma}{2n(1-\gamma)} > \frac{\alpha-1}{\alpha}. \quad (29)$$

At the interior Cournot-Nash equilibrium, each firm's input is determined as

$$l = l^* = \frac{\alpha\gamma(2n-1)(L+L^*)}{2n[\alpha\gamma(2n-1) + 2(1-\gamma)n]}.$$

Substituting this into (29), the home firm's reaction curve cuts the foreign firm's from the above if and only if

$$n < \frac{2\alpha-1}{2(\alpha-1)}. \quad (30)$$

Analogously, the condition that the home firm's reaction curve is less steeper than the foreign firm's is

$$n > \frac{2\alpha - 1}{2(\alpha - 1)}. \quad (31)$$

## Appendix B: the factor price equalization condition

Throughout the paper, we have assumed that good 2 is always positively produced for ensuring unitary wage rates between the countries. This is a convenient assumption but a sufficient condition is needed to justify such factor price equalization. This appendix is devoted to giving such a condition.

See Figure 1 in which Ethier's (1982) type specialization takes place. In the figure, the foreign country specializes in good 1 if the labor endowment point is given between  $DE$  on  $WW'$ . Thus, to exclude this, we need

$$\frac{\alpha\gamma(n-1)(L+L^*)}{n[\alpha\gamma(n-1) + (1-\gamma)n]} < \frac{L^*}{n},$$

where the left-hand side represents the foreign firm's optimal labor input corresponding to  $l = 0$ . This condition is equivalent to

$$\frac{L^*}{L} > \frac{\alpha\gamma(n-1)}{(1-\gamma)n}. \quad (32)$$

See next Figure 3. In the figure, if the endowment is distributed at  $G$ , the resulting trading equilibrium will be  $H$ , which involves the home country's specialization in good 1. Thus, the endowment must be between  $J$  and  $E$  to exclude such a specialization possibility. This condition is given by

$$\frac{\alpha\gamma(2n-1)(L+L^*)}{2n[\alpha\gamma(2n-1) + 2(1-\gamma)n]} < \frac{L}{n},$$

which is equivalent to

$$\frac{L^*}{L} < \frac{\alpha\gamma(2n-1) + 4(1-\gamma)n}{\alpha\gamma(2n-1)}. \quad (33)$$

In sum, the condition for excluding complete specialization in good 1 requires

$$\frac{\alpha\gamma(2n-1) + 4(1-\gamma)n}{\alpha\gamma(2n-1)} > \frac{L^*}{L} > \frac{\alpha\gamma(n-1)}{(1-\gamma)n}. \quad (34)$$

If (34) is satisfied, both countries positively produce good 2 in any trading equilibrium.

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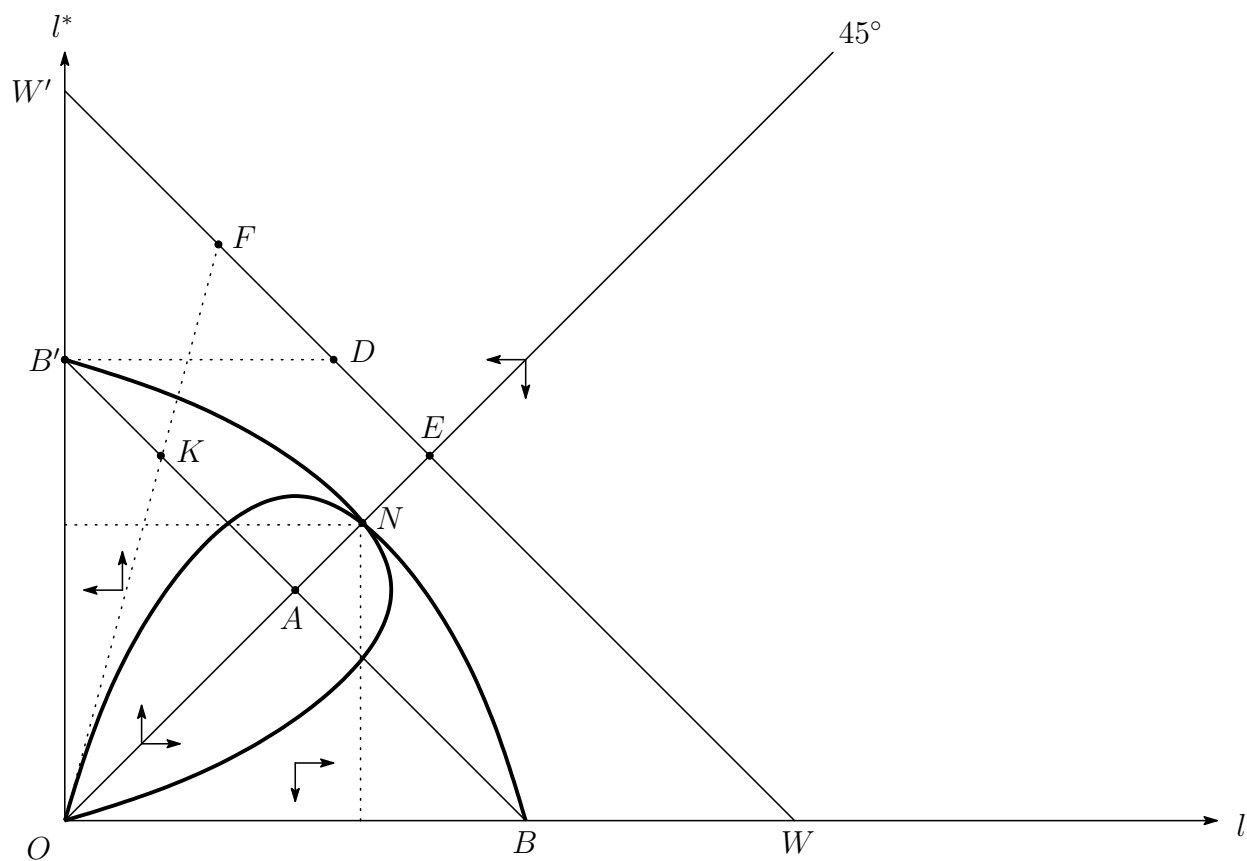


Figure 1: The case with  $n > \frac{2\alpha-1}{2(\alpha-1)}$

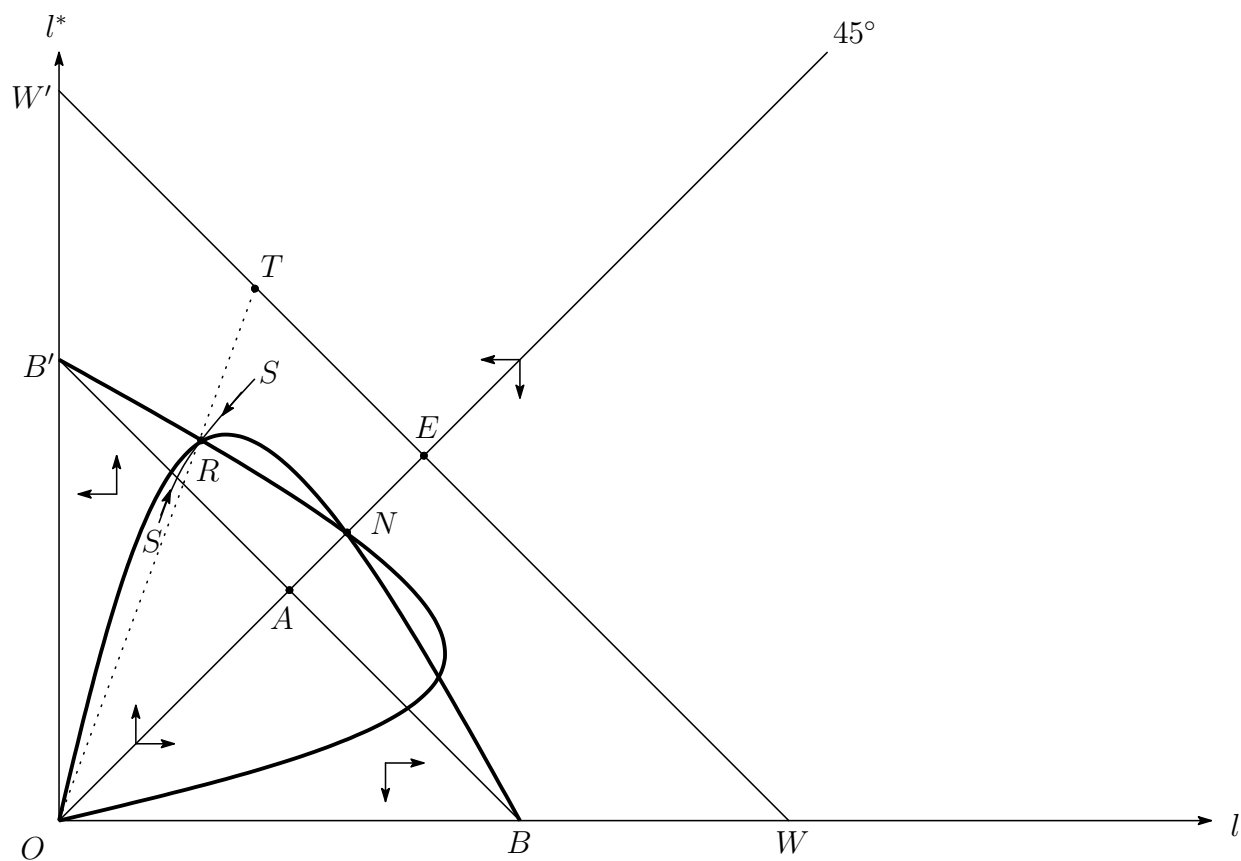


Figure 2: The case with  $\frac{2\alpha-1}{2(\alpha-1)} > n > \frac{\alpha}{2(\alpha-1)}$

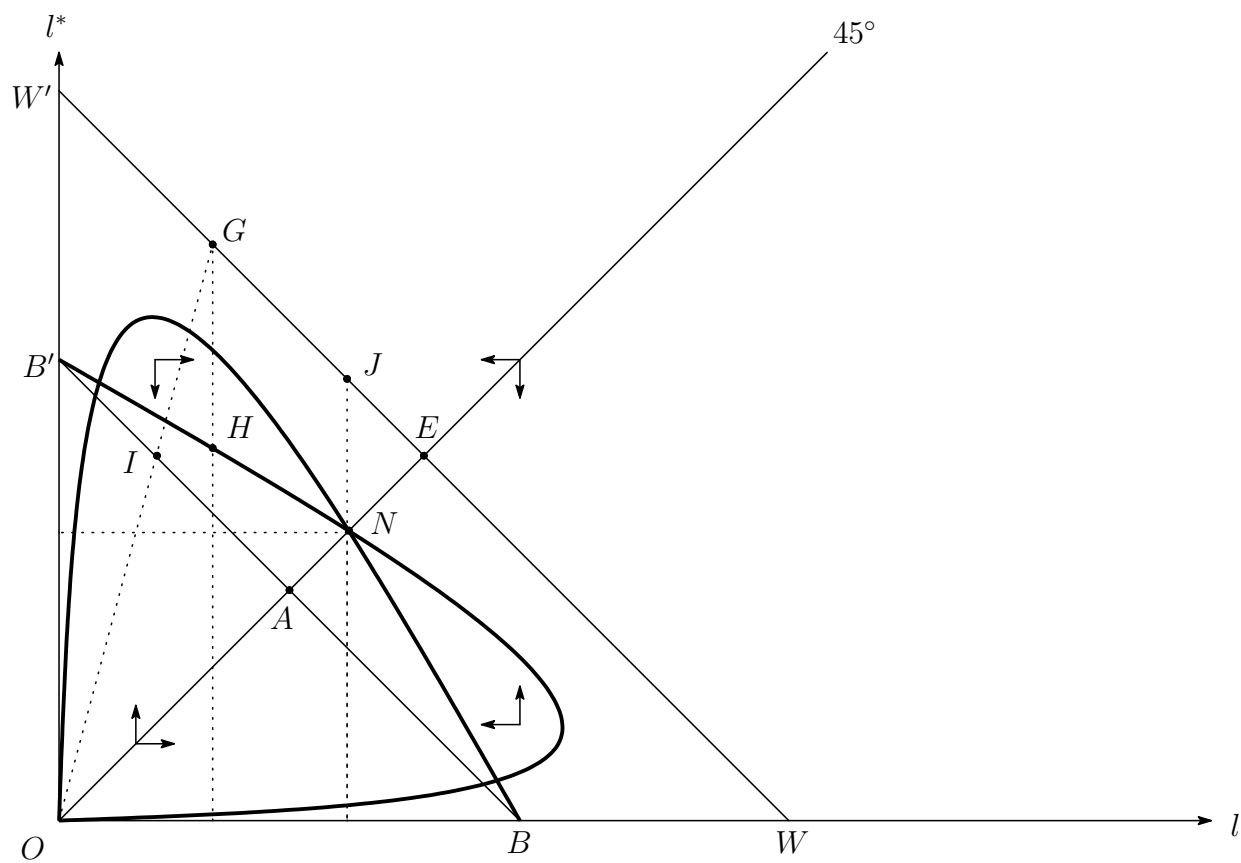


Figure 3: The case with  $n < \frac{\alpha}{2(\alpha-1)}$

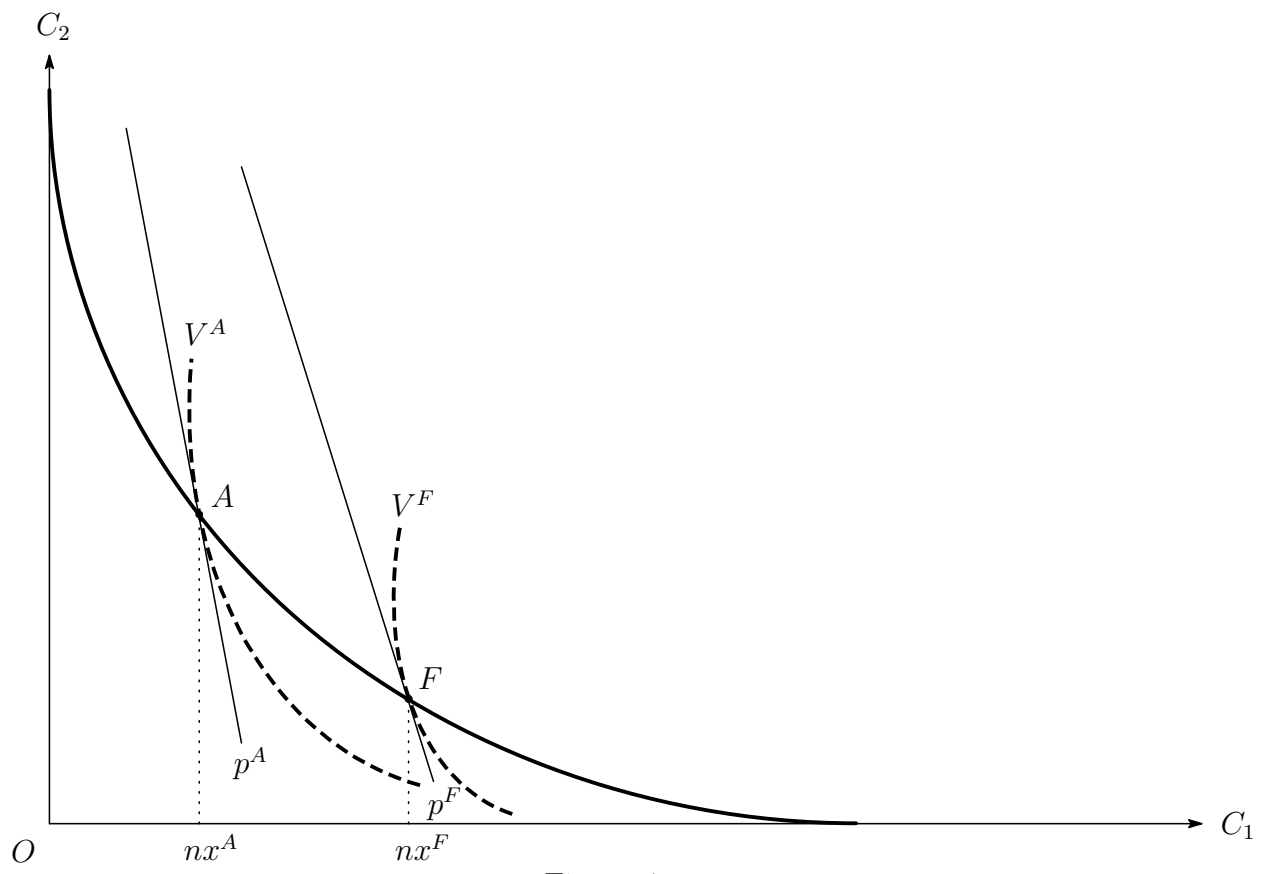


Figure 4: