

## **ALLOCATIVE EFFICIENCY OF A SPECULATIVE FUTURES MARKET WITH ZERO-PATIENCE**

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### **ABSTRACT**

This paper builds on the insight of Gode and Sunder (1993 and 1997) that the imposition of scarcity, not intelligent optimization, is all that is required for allocative efficiency. Using simulated zero-intelligence (ZI) traders in a continuous double auction (CDA) the model of Gode and Sunder (1993) and Gode, Spear and Sunder (2004) is extended to a non-storable commodity futures market with no limit-order book. Replicating the open-outcry on the floor of a futures exchange, trading is random through a stochastic order flow. The exchange sets the trading rules: margin trading and short selling is allowed, real-time gross settlement (RTGS) of margin calls, and a transaction tax on all trades. Prices occur from a sequence of successful market-order trades that match the current bid or ask. Price volatility in this market is determined by the initial distribution of expectations, order-flow and the market microstructure. While this futures market model is dramatically more unstable than the Gode and Sunder ZI stock market model, the results continue to demonstrate the robustness of the CDA to produce prices that converge to Pareto-efficient outcomes, even in a market where agents do not optimize.

**Keywords:** Zero-Intelligence, Margins, Transaction Tax, Continuous Double Auction, Futures Market, Agent-based Model.

JEL Codes: C63, D44, D61

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# 1 INTRODUCTION

The “zero-intelligence” (ZI) methodology, first elaborated by Gode and Sunder (1993) is an extremely innovative methodological tool that attempts to isolate the impact market structure, as opposed to individual cognitive behavior, has in determining price formation in financial markets. It is argued in this methodology that by modeling simple, minimally rational, or random trading behavior, economic analysis can focus on the contribution to price formation from market structure as opposed to agent behavior and learning. This is important since market rules can be designed and optimized whereas individual behavior is inherently private and not directly controllable. This paper reformulates the ZI spot stock market model with a limit-order book, to a speculative futures market with re-trading and no limit-order book<sup>1</sup>. Several other market policy rules are added: short selling, margin trading, transaction costs, real-time-gross settlement, and an endogenous trade size. By incorporating these additional elements it is envisaged that future research could test the implementation and fine tuning of financial markets with the richness of institutional details and test exchange policies through simulation. The

In a two goods market, Gode, Spear and Sunder (2004) show how, in an endowment fixed market, agents with well defined Cobb-Douglas utility functions, who do not profit maximize but possess just enough rationality to stay within their linear budget constraints (or are constrained by market rules to do so), approximate Pareto-efficient outcomes via the rules of a continuous double auction (CDA) mechanism. This was first alluded to by Becker (1962) who analytically showed that in a closed market, rational behavior – the optimization of utility or profit – is not necessary to produce downward sloping excess demand curves for households or firms<sup>2</sup>. Random or irrational agents in such settings can still produce Walrasian or rational market outcomes when wealth is constraining. Similarly, experimental economists who studied auction mechanisms found the decentralized bilateral trading of the CDA to yield a high allocative efficiency (Smith 1962), replacing the need for the Walrasian tâtonnement, central auctioneer, for market clearing. Inspired in part by Herbert Simon’s (1955, 1981) distinction between global rationality and individual satisficing, Gode and Sunder (1993, 1997) were the first to simulate

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<sup>1</sup> Many of the financial jargon are defined in Appendix II.

<sup>2</sup> For example, price changes alter the opportunity set of agents in such a way that even if they choose randomly from this set the expected demand function for a good is inclined to be downward sloping.

random market behavior, creating their *zero-intelligent* (ZI) computer traders, and giving intelligence to the market mechanism rather than to the simulated agents (Miller 2002).

Since this time numerous studies have shown that ZI CDA markets have a very high allocative efficiency and fast convergence to the Walrasian equilibrium. A good review of the ZI literature can be found in Duffy (2006). A significant empirical step forward in this literature came when Farmer et al (2005) showed the ZI model to be surprisingly good at empirical predictions, explaining 96% of the variance of the bid-ask spread, and 76% of the variance of the price diffusion rate (variance of price over time), for stocks on the London Stock Exchange. This suggests that market mechanisms, such as the CDA and the limit order book, may have first order effects on price movements in real markets, over trader rationality and strategy.

The model below builds directly on the ZI model, applying this methodology to the futures market, and adding several significant extensions. In particular, we do not assume well-behaved excess demand functions, but rather adopt a more primitive choice set for speculators – buy, sell or hold, in what ever quantity is possible (set by the market rules). Despite these “perversions” we still find that the market converges to the Walrasian equilibrium through retrading, despite the evolving wealth dynamics and the potential for individual instability among the ZP speculators and their lack of optimizing behavior. The bid-ask spread tends to narrow to the transaction tax when liquidity is available because of the CDA market rules which promote allocative efficiency over time. This paper supports the view of Gode and Sunder that individuals may be impulsive or lack rationality and yet markets can remain orderly and efficient, if the right market institutions are in place.

In the next section we describe the literature that underlies and motivates a ZI futures market model. Our aim, to remove the limit-order book, directly leads to a renaming of our ZI agents into zero-patient (ZP) agents. Rather than no intelligence, they have no patience. It is argued that this is effectively the same as a random decision by a ZI agent with a limit-order book. In section 3 our speculator is specified as a risk-neutral agent who trades on margin and pays a one-way commission or trading cost/tax for each trade. A ZP speculator demand curve is derived with real-time gross settlement (RTGS). The margin requirement, transaction tax, and frequency of settlement are all set by the exchange and imposes market discipline on the speculator. ZPs are endowed with a common amount of wealth (cash), the security is infinitely divisible, and

there is multiple bidding rounds. ZPs are constantly retrading (buying and selling) and quoting bid and ask prices in the CDA. Section 4 presents simulations of the ZP market model with two representative hedgers who set the fundamental equilibrium. We find that the mid-point between the bid and ask converges to the Walrasian equilibrium price and the bid-ask spread narrows to the cost of transacting.

## 2 RANDOM TRADING: ZERO-INTELLIGENCE VS ZERO-PATIENCE

Smith (1982) defines three categories that determine the performance of a micro system: the institutional structure (the rules that govern exchange), the environment (agents' tastes, risk profile and endowments of information and resources), and agent behavior, learning or trading strategy. This last criteria is referred to by Gode and Sunder as "intelligence" and interpreted as rationality or optimization.

Adding to this market structure the paper focuses on the trading of risk-neutral speculators rather than "fundamental" traders. While not relevantly different from Gode and Sunder's (1993) heterogeneous buyers and sellers, they do not have the familiar smooth indifference curves of Gode, Spear and Sunder's (2004) Edgeworth box presentation. By using risk-neutral speculators we hope to not only generalize the results but also add to the potential for market volatility which will aid in the testing of allocative efficiency of the CDA.

In the ZI models of goods or stocks the CDA is able to drive price efficiency with outcomes that closely track the predictions of the Walrasian competitive equilibrium, without the individual optimization criteria usually assumed in the invisible hand analogy<sup>3</sup>. The usefulness of this model allows the ZI research program to aid market design and promote our understanding of the effect market rules have on efficiency, price volatility and liquidity through simulation.

Overall, the ZI traders are a tool to isolate and understand the effect of market rules on market outcomes. Understanding the effects of market rules and other social institutions

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<sup>3</sup> Standard general equilibrium theory requires strong assumptions on agent optimization and information to assure the existence and stability of an equilibrium as in the Arrow-Debreu (1954) model.

is crucial because rules are observable and controllable, while individual strategies are inherently private and not directly controllable. Theories based on the effect of market rules are therefore easier to test. The ZI model provides a benchmark of the “structural” effect of market rules. The traditional strategic model in which traders respond fully to changes in market rules [and price outcomes] is another benchmark. The two benchmarks bracket the range in which human behavior lies (Gode and Sunder 2004, p.2).

The study of the CDA in ZI markets has never been separated from the presence of an order-book. This project removes the order-book (or in other words restricts the limit order book to size one), and changes the market setting from a stock to a futures exchange. Common market structures and rules that dominate futures exchanges are studied e.g. open-outcry, margin trading, short selling, and settlement frequency. This work is a direct extension of research by Ussher (2004) which used a multi-agent model to study the impact of margins and transaction costs on prices in both Walrasian (similar to call auction) and CDA markets with speculators, hedgers and scalpers.

In open-outcry on the floor of a futures exchange, there is no book kept to accumulate limit orders (or rather it can be considered as a market with a limit order book of size one) and the highest bid and lowest ask prevail as the only limit orders available at any one time. In the market model and simulation below, risk-neutral speculators take positions, either long or short, based on their fixed price expectation for the future price. Speculators trade on margin and the exchange covers all counterparty risk. Settlement is effectively contemporaneous although traders queue to trade and meet settlement requirements, and positions are marked to market after each trade. The emphasis, as with Gode and Sunder (1993, and 1997) and Gode, Spear and Sunder (2004), is with the operation of the continuous double auction (CDA) and imposition of “market discipline.” In our case that discipline includes: trading within the budget/margin constraints, forced trading to liquidate a position if wealth does goes below the margin requirement during the imposition of real time gross settlement, transaction costs incurred at each trade, and abiding by the CDA protocol of a futures market.

In an *open-outcry* futures market, as described by Silber (1984), all *bids* and *offers* must be announced publicly to the pit through the outcry of buy or sell orders. In particular, no prearranged trades are permitted on futures exchanges. Strict priority is kept, where the highest bid price and the lowest offer take precedence, and this is known as the *inside spread*. Lower bidders must keep silent when a higher bid is called out, and higher offers are silenced when a lower offer is announced, although simultaneous offers and simultaneous bids at the same price can occur. To increase the probability of execution, a trader can raise his bid or lower his offer, and then other traders must remain silent. This rule is designed to insure best execution, in the sense that sales occur at the highest bid price and purchases occur at the lowest offering, and all bids or offers do not live longer than the moment needed to make a transaction.

It is important to identify the similarities between this speculative futures model and that of Gode and Sunder's (1993, 1994, & 2004) spot or stock market, of constrained random trading. The results here reinforce their same conclusion that the CDA is efficient in converging to the Pareto or Walrasian equilibrium without trader optimization and, in addition, even when potentially destabilizing leveraged speculators exist.

In the two good model below, of cash and future contracts, agents are risk neutral and speculate on future price changes. Illiquidity is characterized by the degree to which speculators are leveraged and the enforcement of real-time gross settlement (RTGS) which requires speculators to mark to market their position after every trade. Transaction costs of 0.1 percent on one-way trades and a 25 percent margin requirement add to the characterization of illiquidity in the market. Speculators will go long or short in their derivative position based on their fixed expectation of future prices. Unlike most agent-based models (ABMs) (see summary of ABMs by Tesfatsion 2002) there is no evolutionary process in agent behavior, but there is an evolution wealth distribution. In this manner this 2 good model is closer to the general equilibrium ZI models of Gode, Spear and Sunder (2004) where wealth is explicitly modeled, as opposed to their original partial equilibrium ZI model of Gode and Sunder (1993).

The Gode and Sunder ZI constrained model (1993) has traders divided into buyers and sellers who are given an equal allotment of shares to buy or sell, respectively. Each buyer (seller) has a given resale (cost) value remains fixed and creates the upper bound of their budget constraint.

All traders maintain their desire to trade until they reach their pre-specified allotment of securities, which is the same for all traders. Their participation is not guided by optimization - minimization costs (maximize of profit) (which are calculated at the end of the trading period). Rather, all traders offer simultaneously a random bid (ask) that ranges between some nominated floor (ceiling) and their budget constraint. Traders are selected and their bid (ask) for a single unit of the good is submitted to the limit order book where they are ranked, in accordance with CDA rules, such that the highest bid (lowest ask) is considered as the current bid (current ask). A trade occurs when the new bid equals or exceeds the current ask (the new ask equals or is less than the current bid). Following a transaction the limit order book is cleared and a new round of bids and asks are solicited and the process is repeated.

In contrast, the model presented here simulates liquidity constrained speculators who trade futures contracts and have fixed uniformly distributed expectations on the future spot price. The zero-intelligence characteristics are thought to be preserved, but the stock model is now applicable to a derivate model where promises rather than goods are traded for cash. In the simulations below the futures market is for a non-storable commodity and the spot market is independent of the futures trading, and is ignored in this trading period.

All speculators start with an equal endowment of cash to buy or sell futures contracts<sup>4</sup>. The speculators' risk-neutrality and marked to market wealth makes the decision – buy, sell, or hold<sup>5</sup>. A speculator expects to make a profit by buying low and selling high, relative to their spot price expectation<sup>6</sup>. Speculators will maintain their desire to buy or sell until constrained by their wealth, transaction costs and margin requirement. As in all futures markets, all long plus short positions sum to zero. Open outcry, an oral CDA, is used to find the current bid and ask, which is technically equivalent to a limit order book of length 1. Speculator participation in the market

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<sup>4</sup> Gode and Sunder (1993) only has one good with no wealth restrictions, and Gode, Spear and Sunder (2004) have two goods: green and red chips. This model has two goods, cash and futures contracts, but the derivative has a zero-sum in aggregate. Transaction costs are a leakage from the system.

<sup>5</sup> In Gode and Sunder, traders are selected to be either buyers or sellers hence producing an aggregate demand curve and an aggregate supply curve. Here, speculators produce just an aggregate demand curve that has both positive and negative sides. This is technically a “total demand” curve, but it looks similar to the “excess demand” curve of general equilibrium analysis.

<sup>6</sup> Just as the resale or cost valuations were fixed in Gode and Sunder (1993), the expectations of the underlying commodity valuation in the future spot market is fixed, and it is initially drawn from a uniform distribution.

is guided by an *immediacy* or impatience to transact using market orders, rather than the Gode and Sunder random limit orders that are kept in the limit order book. It is argued here that both strategies are equivalent, and that both omit intelligent optimization strategies. As with Gode and Sunder's emphasis on market discipline, speculators in this model are constrained from accumulating losses by obeying margin calls settled in real-time.

Both models outlined above incorporate zero intelligence and budget constrained traders in an attempt to determine whether it is intelligence or market discipline that is most important for the process of price discovery to the Pareto optimum. Gode and Sunder test 3 CDA markets: human constrained, ZI-constrained, and ZI-unconstrained; finding that the first two markets are almost identical in their allocative efficiency, thereby concluding that it is the constraints within a CDA that was responsible for the allocative efficiency of the market and not intelligence. This paper compares the multi-lateral Walrasian equilibrium (super intelligence) to the ZI-constrained CDA price. While the traders here are ZI-C, they are renamed as zero-patience (ZP) traders to distinguish their trading with market orders rather than limit orders.<sup>7</sup>

ZP speculators desire *immediacy*, trading primarily with market orders when there is an expected gain, or the need to meet real-time margin requirements. ZP speculators are satisficing rather than optimizing, and in this sense are void of learning and intelligence. The impatient trading behavior specified here for speculative traders mirrors the random behavior of fundamental traders in the Gode and Sunder (1993) ZI stock market model. Both are subject to budget constraints and market rules but both typically trade at prices that do not optimize their profit. The probability of placing a market order at a certain quoted price in this model is similar to having a limit order accepted in the Gode and Sunder model due to the common CDA architecture in both models.

ZP speculators are randomly selected to trade in rounds and will submit a market order to buy at the current ask (sell at the current bid) if their end-price expectation is higher (lower) than this value. Because expectations are randomly distributed the probability of trading at the current bid or ask should be similar to the probability that Gode and Sunder's agents trade following their

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<sup>7</sup> Zero-intelligence/patience (ZIP) traders may have been an alternative, but this anachronism has already been used by Cliff and Bruten (1997) for their zero-intelligence-plus traders.

random quotes. If the speculator's end-price expectation is within the current bid-ask spread then they will not transact, but instead better one of these quotes, replacing either the current bid or ask with their own and thus narrowing the spread, as in Gode and Sunder. The details of this model will be elaborated on in section 4.

### 3 A MODEL FUTURES MARKET WITH SPECULATORS AND HEDGERS

#### 3.1 Trader Population

This futures market model includes two different traders: speculators and hedgers. They contribute to the price formation in different ways, but in both cases trading behavior focuses on quantity and wealth constraints rather than strategies that optimize utility.

We present a model of a futures trading pit with *open-outcry* and a continuous double auction trading mechanism of bilateral exchange and limit-order book. The model has two markets: a speculative futures market for an underlying non-storable commodity<sup>8</sup> and a residual money market whose price is normalized to 1. One-way transaction costs and margin requirements, are calculated as a percentage of the market value of each trade or position respectively, and imposed by the exchange on all traders [even hedgers?]. There is no restriction on short selling, the futures contract size is perfectly divisible, and prices are always non-negative. Traders attempt to mark-to-market their position at every trade.

While real time gross settlement (RTGS) is an exchange rule and traders are compelled to try to settle this with the exchange before each trade takes place through *variation margins*<sup>9</sup> this may at

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<sup>8</sup> The non-storability of this commodity, characteristic of climate or electricity futures, allows us to ignore the possibility of arbitrage between the spot and futures market. In this model the spot market which is closed during our trading period. Extensions of this paper will include commodity intermittent spot trading.

<sup>9</sup> Instead of using the close-of-day settlement price to calculate margin calls, settlement is adjusted continuously throughout the day and the settlement price used to calculate margin calls is the average of the bid and ask price, or last mid-price. This means that profits and losses transfer hands via the exchange, between traders continuously, removing the risk of accumulated losses and trader default. The amount paid is known as the *variation margin*. As a simplification, *initial margin* and *maintenance margin* are consolidated into a margin requirement which is

times only be accomplished after trading: speculators must wait their turn to finalize a bilateral trade if a margin call requires liquidation of their position. As in all futures exchanges there is no counterparty risk: bankrupt traders are allowed to bring their negative position back to zero through trading and then halt all trading activity. In effect a bankrupt trader's position is taken over by exchange. There is no replacement of bankrupt traders in this version of our model.

Each type of trader has their own quantity constraints and rules for trading. *ZI speculators* will trade, buy or sell, for expected capital gains. All speculators are risk neutral and differ only in their expectation of what the futures price should be and their cumulative wealth positions. Expectations of the fundamental futures price (futures spot price) stays constant for each speculator during the trading period. Being risk neutral, trading on margin and trading long or short at every chance they get to trade, speculators typically end up at the corner solutions of their budget constraint over time. Due to the absence of a limit order book, the *ZI speculators* in this model are characterized as *zero-patient (ZP)* traders. If a trader has a price expectation that is within the inside bid-ask spread then they will replace the current market quote with a better limit order. But in most cases, all speculators will enter the market with either feasible or non-feasible market orders. Since market orders are known as immediacy trades, we modify the term from zero-intelligence to zero-patience. It is our belief that this modification retains the essential randomness of the original *ZI* model given the random distribution of reservation prices of our speculators.

*Hedgers* play a limited but important role in setting up the fundamental demand and supply of contracts in the market. There are only two representative hedgers — one going long (to lock in the purchase price of the fixed quantity of the underlying commodity, in the future spot market) and the other going short (to lock in the sale price for the desired quantity of the underlying commodity in the future spot market). Hedgers place only market orders until they fill their exogenous desired contract position. Once their futures position is attained, they stop trading. The difference between the long and short hedge is the *net hedge*, or the net desired contract

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specified as a fixed percentage of the contract value rather than an absolute dollar value per contract as occurs in security futures. By using a margin requirement that changes with the percentage change in prices, we better approximate the essence of what the exchange considers in setting the margin.

position of the hedgers. Since the sum of all futures contracts sum to zero, the net hedge will determine the long-run net position of the remaining trader population<sup>10</sup>.

The net hedge is used here to counter the Cliff and Bruten (1997, p.19) criticism of the Gode and Sunder (1993) ZI model. They replicated the ZI constrained model and found that the mean trading price for the ZI traders was only close to the theoretical equilibrium price when the reservation prices averaged around the equilibrium price and the supply and demand curves were symmetric. When the demand and supply functions were not symmetric, Cliff and Bruten (1997) showed that the agents required some intelligence and they created ZI-plus traders which incorporated an adaptive behavior based on the behavior of other traders. However, this is an unnecessary addition of intelligence. Rather, imposing market discipline with a closed market of goods (in a general equilibrium context) is enough to bring about a convergence to the theoretical Walrasian equilibrium price. This was shown by Gode, Spear and Sunder (2004) and it is supported in the results below where we have simulated 2 asymmetric markets with a positive net hedge of 5000 contracts.

### 3.2 ZP Speculator's Demand Function

In our model with leveraged speculation,  $\kappa$  represents the limit on how much larger a speculator's futures position — price multiplied by the number of contracts ( $p_t x_t$ ) can be than a trader's wealth  $m_t$ . All simulations in this paper use  $\kappa = 4$ , which means that a trader can have up to 4 times his wealth dedicated to a long or short futures position. In other words the margin requirement is 25%,  $1/\kappa = 0.25$ . The collateral kept in the margin account by speculator  $i$  is held as Treasury bills or money<sup>11</sup>, represented here as  $m_t^i$ . Collateral held must be greater than the margin requirement,  $m_t^i \geq p_t x_t^i / \kappa$  for the current futures position, at all times (to the extent that trading allows). There will be several transaction prices through out the day which represent a trade at either a quoted bid  $p_t^b$  or a quoted ask  $p_t^a$ . If there is not enough collateral in the

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<sup>10</sup> Keynes argued that the net hedge is typically negative and justifies the backwardation of futures prices.

<sup>11</sup> There is no opportunity cost in holding cash since it would receive an interest on the T-bills, however interest in this version of our model is zero.

margin account to meet the margin requirement then speculator  $i$  will have to liquidate their position with an offset purchase or sale at their next turn to trade.

The futures position  $x_t$  at price  $p_t$  is taken on by the speculator as a contract at time  $t$  to sell or buy  $x$  units of the underlying commodity at price  $p_t$  on the spot or maturity date of the futures contract. Since the speculator does not intend on making delivery on this contract, the purpose of holding this position is to flip the position and profit on price changes. On the basis of a constant price expectations  $p^{i,\theta}$  about the next transaction price  $p_{t+1}$ , speculator  $i$  will decide to go either long or short in futures. Transaction costs are incurred for each one-way trade as a percentage of the trade value. In the simulations below this fee 0.1 percent and is represented by  $\varpi = 0.001$ . If the expected short-term gain does not compensate the cost of trading over the next period:

$$(p^{i,\theta} - p_t) x_t \leq \varpi p_t | (x_t - x_{t-1}) |$$

then the speculator will hold his current position instead of trading. The trader is myopic and on opening a position there is no consideration of costs incurred for reversing the position. Each speculator  $i$  is risk neutral and simply maximizes expected wealth  $\pi$  over the period  $t$  to  $t+1$ :

$$\pi_{t+1}^{i,e} = (p^{i,\theta} - p_t) x_t^i + m_t^i$$

The speculator's demand curve is derived in Appendix 1 via linear programming. In summary, speculator  $i$ 's demand for futures in each period  $t$  is a slightly simplified version from Ussher (2004):

$$x_t^i(p_t; x_{t-1}^i, m_{t-1}^i, p_{t-1}, p^{i,\theta}, \kappa, \varpi,)$$

where:

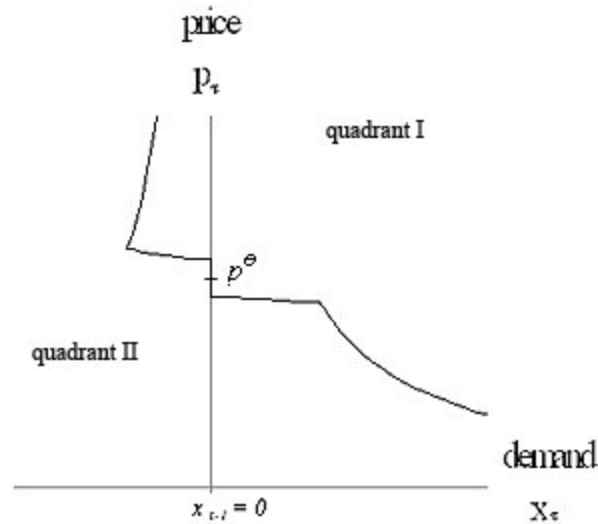
$p_t$  = Intra day futures market transaction price at time  $t$ ,

$x_{t-1}^i$  = Previous contract position,

- $m_{t-1}^i$  = Previous cash position in margin account following last transaction,
- $p^{i,\theta}$  = Price expectation  $p^\theta$  of the next futures price  $p_{t+1}$ ,
- $1/\kappa$  = Margin requirement as a percentage of futures position value, and
- $\varpi$  = Percentage transaction tax on a one-way trade (paid each way).

A futures demand curve is usually represented as a smooth downward sloping line from the top of quadrant II to the bottom of quadrant I in the two dimensional  $R^2$  space in Figure 1. This model produces a non-linear demand function with inherent corner solutions from the risk neutral speculators' wealth constraints and the regulatory setting of margin limits, transaction costs and RTGS.

Each risk-neutral speculator maximizes the next period's expected wealth by holding money as collateral and buying or selling (going long or short in) futures. The decision to buy or sell futures depends on whether the speculator expects prices to rise or fall, respectively. There is no restriction or disincentive to *short selling* (i.e., selling commodities that one doesn't own). A trader will trade only when price expectations  $p^\theta$  are far enough away from actual prices  $p_t$  to pay for the one-way transaction costs. Figure 1 has a zero contract position held over from last period. If a speculator currently has a futures position, then margin calls can lead to forced liquidation of the position when prices move against expectations. The possibility of a backward bending demand function, as in Figure 2, is a result of the collateral  $px$ , which underlies demand for  $x$ , being priced in the same market.



**FIGURE 1** A speculator's demand for futures  $x_t$  as a function of  $p_t$ , with a past zero position  $x_{t-1}$ , and price expectations of  $p^e$

The speculator will sell (buy) futures if he expects the price to fall (rise) when the slope of the demand function is positive. The demand function has a negative slope when purchasing power is declining from higher futures prices or when collateral is devalued and the speculator must liquidate part of his position to maintain the margin requirement.

At each  $t$ , the variation margin is calculated and net wealth is adjusted. The mid-price  $p^m$  is the average of the bid quote  $p^b$  and ask quote  $p^a$ :

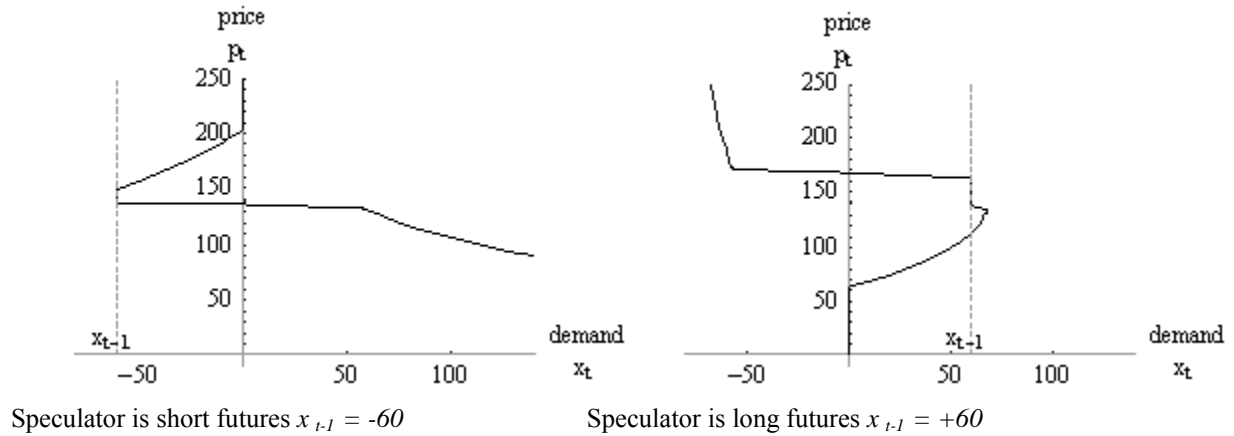
$$p^m = (p^a + p^b) / 2$$

The profit or loss is calculated with price changes of the mid-price and paid from the losing agent to the winning agent via the exchange clearing house, equivalent to

$$(p_t^m - p_{t-1}^m) x_{t-1}^i$$

Each speculator estimates his net wealth at each  $t$  given prices  $(p^a, p^b, p^m)$  which determines their decision on how many futures contracts to buy or sell to maximize expected wealth, while at the same time meeting his margin requirement — a position,  $p_t x_t$ , that is less not more than net

wealth multiplied by  $\kappa$ . The mid-price is used in accounting for net wealth every period, as long as a position is held.<sup>12</sup>



**FIGURE 2** A speculator's demand curve with either a short or long starting position:  $m_{t-1} = 5000$ ,  $p^\theta = 150$ , and  $\kappa = 2$  for each graph

### 3.3 The Bidding and Trading Process

Within the CDA, speculators are selected randomly for a sequence of bilateral trading with non-replacement in each round, so that each trader has an equal chance of trading and trades every round. The hedgers are placed last in this sequence, which represents one round. The intraday period of futures trading has several rounds of *quoting* or *transacting*, at the bid or ask price. Quantities traded and their transaction prices are registered at each time  $t$ .

Central to the trading process is the auction that simulates the open-outcry on the floor of an exchange, leading to transactions and thus transaction prices. It is a groping mechanism where

<sup>12</sup> After the initial purchase of a market order the trader must pay a *variation margin* of  $(p_t^m - p_{t-1}^m)(x_t^i - x_{t-1}^i)$ . Important in this calculation of variation margin is that we keep the distinction between those that profit by buying at the bid or selling at the ask, versus those who are considered impatient and sell at the ask or buy at the bid. When a contract is bought and  $(x_t^i - x_{t-1}^i) > 0$ , if it is bought at the bid with a *limit order* then the variation margin is positive  $(p_t^m - p_t) > 0$ . If however it is bought at the ask with a *market order* then the variation margin is negative  $(p_t^m - p_t) < 0$ . This results in a transfer of wealth from the trader who is willing to pay for *immediacy* to the trader who gets paid for providing liquidity and *making the market*. The maximization of expected wealth by the speculator only takes into account the expected change in the trade price  $(p^\theta - p_t)$  without anticipating whether the transaction is by market order or limit order.

both bid and ask prices adjust and where out-of-equilibrium trades take place when an agent agrees to sell contracts (hit the bid) to another agent who is bidding for them, or when another agent decides to buy contracts (lift the ask) from the agent who is asking for them. This process of quoting and trading is repeated many times, giving each market participant the chance to quote and trade several times and fill his orders. No new information is brought into this process; expectations remain constant.

The competitive bidding algorithm presented here for the ZP speculators is drawn from several sources. The manner in which speculators compete and how their price expectations interact with the bid-ask spread during the bidding process comes from Chan et al. (1998) and Yang (2002). An important modification to their model, apart from keeping expectations constant, is the presence of risk-neutral speculators with collateral constraints and transaction costs. Hedgers act similarly to speculators but only place market orders to fill expectations and do have constraining margin requirements. Hence, hedgers do not compete in the bid-ask spread.

This asynchronous bilateral bidding process allows two or three traders to participate at any one time: offering, or bettering, limit order quotations or carrying out market order trades. Speculators take turns in entering into the inter-dealer market to quote price and quantity, to transact, or to exit. A round is completed when all agents have participated once, with the hedgers coming last. This is repeated for a different random sequence of speculators for more than 50 rounds. The repetition of trading rounds represents competition within the price mechanism and helps the convergence to equilibrium of market demand and supply. This bidding and bilateral trading process is detailed next.

However, when a market order “crosses” the inside bid and ask limit orders there is a chance that the spread will widen

### **3.4 Auction Algorithm for a ZP Speculator**

ZP speculator  $i$ 's reserve price is his expected price,  $p^{i,\theta}$ , plus or minus the one-way transaction tax  $\varpi p_t$ . Half of the bid-ask spread is often thought of as a measure of the cost of executing a

market order (the difference between the mid-point price and the transaction price). We shall represent this half spread by a lowercase letter  $s$ . The size of  $s$  is endogenous to the bilateral trading process. It should converge to the percentage transaction cost in a set of compact expectations. For example, a price expectation that lies within the spread will narrow the bid ask spread by offering a competing order. The new limit order (bid or ask) is chosen by adding or subtracting the current spread to the price expectation ( $p^{i,\theta} \pm s$ ). This will create a single competing limit order, and this one becomes the new bid or ask, thus narrowing the spread approaching  $\varpi p$ .<sup>13</sup>

If on the other hand there is no bid or ask (a zero quantity bid or zero quantity ask), a speculator will announce his own noncompetitive limit order and increase the half spread on the basis of expectations by a significant amount  $(I \pm S \varpi) p^{i,\theta}$ . In this case,  $S$  is a multiple of the transaction fee. If  $S$  is greater than 1 then the new limit order will guarantee that a new hit or bid occurs with a demand different from zero. In our simulations we use  $\varpi = 0.1\%$  and  $S = 8$ , which means that the new bid is within a 1% difference from their price expectation.

We present the trading algorithm with three traders: inter-dealers  $k$  and  $j$  (which could be a speculator or scalper) and a new ZP speculator, entrant  $i$ . In this presentation, agent  $i$  represents a speculator who demands immediacy and will always prefer to trade with a market order rather than a limit order if possible. ZP speculator  $i$  enters the market and witnesses the current bid:ask ( $p^a : p^b$ ) spread and makes a trade choice under the following four scenarios.

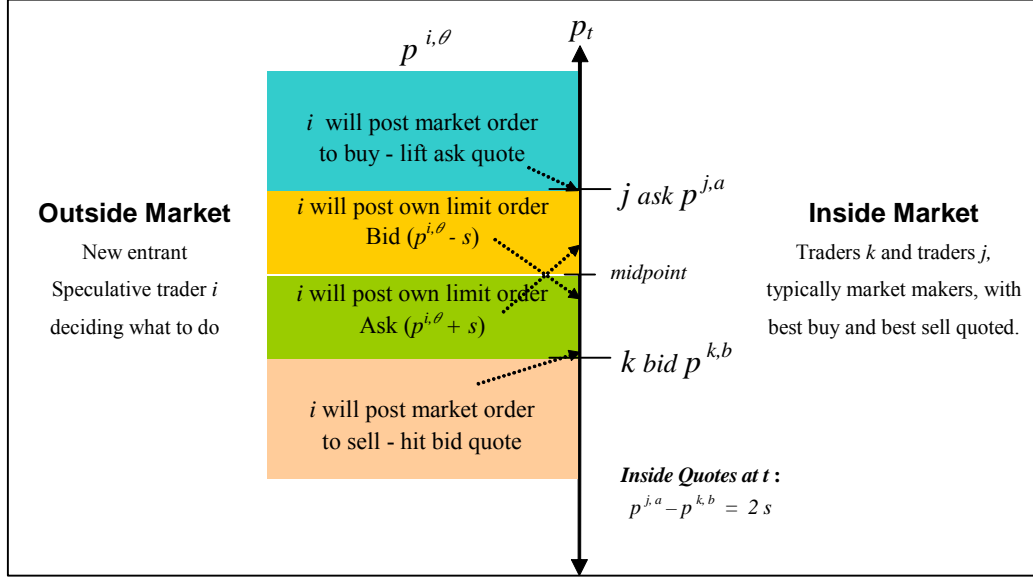
- Scenario 1. (Figure 3a) The ask,  $p_t^{j,a}$  and bid,  $p_t^{k,b}$ , currently exist with non-zero offers, at time  $t$ .
  1. If  $p^{i,\theta} > p_t^{j,a}$  speculator  $i$  will post a market order and buy at this ask price — lift the ask quote.
  2. If  $p^{i,\theta} < p_t^{k,b}$ , speculator  $i$  will post a market order and sell at this bid price — hit the bid quote.

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<sup>13</sup> This rule may be said to be “optimizing” but it is considered here simply taking the accepted “commission.”

3. If  $p_t^{k,b} \leq p^{i,\theta} \leq p_t^{j,a}$  and  $< (p_t^{k,b} + p_t^{j,a})/2$ , speculator  $i$  will post a sell limit order at a price of  $(p^{i,\theta} + s)$  and thus quote his own ask, replacing agent  $j$

4. If  $p_t^{k,b} \leq p^{i,\theta} \leq p_t^{j,a}$  and  $\geq (p_t^{k,b} + p_t^{j,a})/2$ , speculator  $i$  will post a buy limit order at a price of  $(p^{i,\theta} - s)$ , and thus quote his own bid, replacing agent  $k$



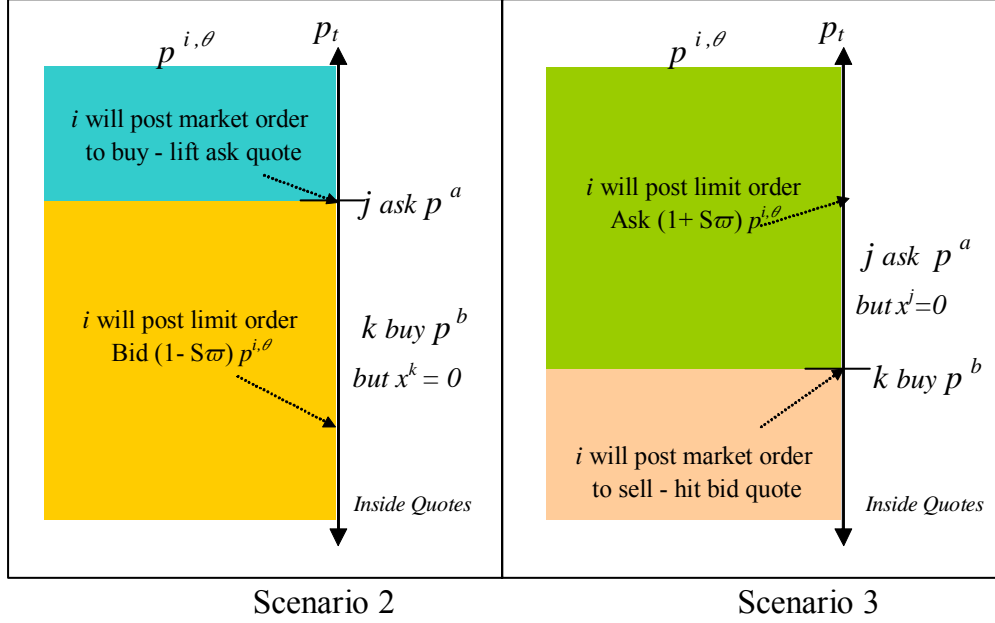
**FIGURE 3a** Scenario 1, in which both competitive quotes — bid and ask — exist in the marketplace prior to new entrant

- Scenario 2. (Figure 3b) Only the best ask,  $p_t^{j,a}$ , exists; that is, at  $p_t^{k,b}$  demand to go long is zero as  $(x_t^k - x_{t-1}^k) \leq 0$ .

1. If,  $p^{i,\theta} > p_t^{j,a}$  speculator  $i$  will post a market order, buy at this ask price.
2. If  $p^{i,\theta} \leq p_t^{j,a}$ , speculator  $i$  will post a buy limit order  $p_t^{i,b}$  at a price of  $(1 - S \varpi) p^{i,\theta}$ , but only if excess demand at this price is  $(x_t^i - x_{t-1}^i) > 0$

- Scenario 3. (Figure 3b) Only the best bid,  $p_t^{k,b}$  exists; that is, at  $p_t^{j,a}$  demand to go short is zero as  $(x_t^j - x_{t-1}^j) \geq 0$

1. If  $p^{i,\theta} < p_t^{k,b}$ , speculator  $i$  will post a market order and sell at this bid price;
2. If  $p^{i,\theta} \geq p_t^{k,b}$ , speculator  $i$  will post a sell limit order  $p_t^{i,a}$  at a price of  $(1 + S \varpi) p^{i,\theta}$ , but only if excess demand at this price is  $(x_t^i - x_{t-1}^i) < 0$



**FIGURE 3b** Scenario 2 in which an ask but no bid exists prior to new entrant and Scenario 3, in which a bid but no ask exists prior to new entrant

- Scenario 4. If no bid or ask effectively exists; that is at the ask quote  $p_t^{j,a}$ ,  $(x_t^j - x_{t-1}^j) \geq 0$ , and at the bid quote  $p_t^{k,b}$ ,  $(x_t^k - x_{t-1}^k) \leq 0$

1. The new entrant speculator will post both a buy and a sell limit order at  $(1-S\varpi)p^{i,\theta}$  and/or  $(1+S\varpi)p^{i,\theta}$  respectively, as long as his bid is quoted for a buy of greater-than-zero contracts, and the ask is to sell greater-than-zero contracts. If this is not the case then the current bid-ask remains, even though both traders have zero demand, and entrant  $i$  exits to join the queue to trade again later.

In this model, under Scenario 2 (Scenario 3) the speculator tendering the best bid (ask) might have had prices move against him; for example, if he is long (short) and prices fell (rose). They may remain offering a bid (ask) price to buy (sell), but at a quantity of zero. Now he wants to offset his position and sell (buy) so that excess demand is less (greater) than zero.

$$\text{Scenario 2: } (x_t^k[p_t^{k,b}] - x_{t-1}^k) \leq 0 \quad \text{where } x_t^k \text{ is a function of } p_t^{k,b}$$

$$\text{Scenario 3: } (x_t^j[p_t^{j,a}] - x_{t-1}^j) \geq 0 \quad \text{where } x_t^j \text{ is a function of } p_t^{j,a}$$

Effectively under Scenario 2 (Scenario 3) agent  $k$  (agent  $j$ ) falls silent and will eventually be replaced by a new entrant, as long as the new entrant has  $p^{i,\theta} < p_t^{j,a}$  (has  $p^{i,\theta} > p_t^{k,b}$ ) and as

long as  $[x_t^i(1 + S\varpi)p_t^{i,\theta} - x_{t-1}^i] > 0$  (as long as  $[x_t^i(1 - S\varpi)p_t^{i,\theta} - x_{t-1}^i] < 0$ ) otherwise agent  $k$  (agent  $j$ ) will remain. Only when agent  $k$  (agent  $j$ ) is replaced and exits the market will he be given the chance to satisfy margin requirements by liquidating their position with a market order, in turn, in the random trading round.

This model considerably changes the Chan et al (Ibid) rules, which emphasize the manner in which price formation feeds back into the market by agents updating their expectations, to one where price formation feeds back into the market via quantity constraints, margin requirements, and inventory control. The model allows for leveraged trading and short selling and makes the method of settlement a central variable of the model.

### 3.5 Auction Algorithm for a Representative Hedger

Hedgers are only concerned about filling their expected sales or purchases at the spot date via market orders in futures. They always come last in each round of the random sequence of speculators and scalpers.

- Hedger Scenario:

1. The future purchaser of the commodity at spot, agent  $q$ , will lift the ask,  $p_t^{j,a}$ , for maximum ask quote quantity, in each round until market buy order is filled  $x_t^q = x^{q*}$
2. The futures seller of the commodity at spot, agent  $r$ , will hit the bid,  $p_t^{k,b}$ , for the maximum bid quote quantity in each round until market sell order is filled  $x_t^r = x^{r*}$

Since speculators and scalpers do not usually offer large size limit order contract lots, it may take several rounds for each hedger to finalize their purchases or sales. The hedgers contribute so called *fundamentals* to the speculative market.

### 3.6 The Trading Sequence

In the simulations which follow, trading begins with a random ordering of 20 speculative agents all with \$10,000 cash. The two representative hedgers come last in this sequence or trading round. Speculators have equal endowments and the heterogeneous expectations are taken from a uniform distribution  $p^\theta \sim U(100,130)$ . Speculators come together, along with hedgers and scalpers, for a length of 500 bilateral trades and prices. The quantity traded is the lesser of the market and limit order demands that are crossed in the CDA.

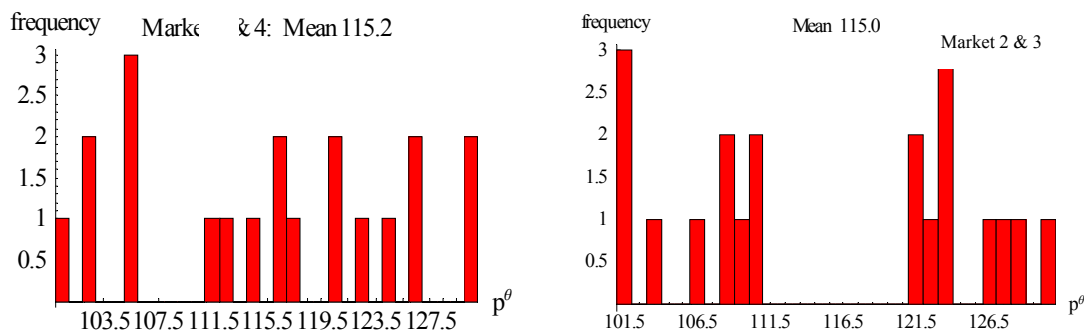
Two randomly selected traders speculators begin with initial random market quotes, set 10 points apart, e.g.  $p_0^b = 120 : p_0^a = 130$ . A new entrant, randomly selected from the remaining traders (but not a hedger), enters the floor to either accept or better the prices quoted. If a bid or ask is accepted, a trade is done and a transaction price  $p_t$  occurs for the market order by the new entrant. If, instead, the entrant replaces a bid or ask or both, then a new set of quotations  $p_1^b : p_1^a$  (bid:ask) is created, with no transaction price. A sequence of quotes, and transaction prices, is generated as each agent enters the market during the trading round, with only transaction prices and volumes registered. Repeating the round, drawing a new random sequence of speculators each time, creates an inter-day trading session. This trading sequence is summarized here:

1. Speculators are initialized with initial wealth and random price expectations. Two randomly selected speculators or scalpers begin with initial quotes of  $p_0^b : p_0^a$  and their respective buy and sell quantities (which may be zero), given their expectations.
2. The random sequence of speculators entering the market, with non-replacement, is determined, with hedgers coming last.
3. With one or two agents quoting a bid-ask spread, the new entrant can either submit a new bid or ask, accept the existing bid or ask, or hold (pass).
4. A transaction occurs when the existing bid or ask orders are accepted and the transaction price is recorded accordingly. The transaction is the minimum of the quantities proposed for exchange by each bilateral trader.
5. At each point, mid-point prices are used to calculate speculator budget constraints in real time. On the basis of the past transaction price, each agent's wealth is updated, taking account of all margin calls (profits and losses).

6. Steps 3 through 5 are repeated for  $n$  times,  $n =$  number of traders (one round).
7. Steps 2 through 6 are repeated for  $N$  times,  $N =$  number of rounds.
8. Final market price is recorded as the 500th transaction price for this trading session.

#### 4 SIMULATIONS of CDA TRADING CONVERGENCE

We run simulations for a 25 percent margin requirement and 0.1 percent transaction cost for two different distributions of speculator expectations, and two different initial conditions for the net hedge position, producing 3 market scenarios. The ZP speculators have expectations taken from a uniform distribution  $p^\theta \sim U(100, 130)$  in market 1 and a binomial distribution in markets 2 and 3 of  $p^\theta \sim \{U(100, 110), U(120, 130)\}$ . The realized frequency distributions of these two ZP speculator expectations are fixed for each scenario and displayed in Figure 4. By comparing two different sets of expectation dispersion we hope to see what impact this may have on the bid-ask spread and price volatility.



**FIGURE 4** Two different distributions of the ZP speculator expectations.

In each run we have a relatively small market<sup>14</sup> of just 22 traders: 2 representative hedgers and 20 ZP speculative agents all starting with a \$10,000 cash endowment. In markets 1 and 2 the *ex ante* net hedge is set to zero and in the other market it is set to an excess demand of 5000, which means that on net the hedgers want to be 5000 futures long. In all cases the hedgers support the fundamental equilibrium price, since speculators as a group have approximately zero demand in aggregate.

<sup>14</sup> The market size is similar to that in Gode and Sunder (1993) and Gode, Spear and Sunder (2004).

Each trading round begins with a random sequence of speculators' and their limit orders (based on their expectations) and then hedgers entering with market orders at the end of each round. Bid, ask and transaction prices are produced through bilateral negotiations. At the end of each round a new one begins with a new random ordering of speculators and hedgers. Trading continues until  $t$  bilateral trades or transaction prices are produced at which time trading ends. In all markets  $t$  is set to 500. In the examples below approximately 1000 to 2000 trading rounds are required to produce 500 transaction prices. The quantity traded in each transaction is the lesser of the market and limit orders that are crossed (hit or lifted) in the CDA. Since there is no exogenous limitation on trade size, the size of an average trade gets smaller and smaller as  $t$  increases and the trading surplus is distributed. In comparison to Gode and Sunder (1993), or Gode, Spear and Sunder (2004), our flexible trade size better highlights the contribution of the CDA process to market clearing and price convergence to the Walrasian equilibrium.

Table 4 (attached) has some exemplary results for the market scenarios 1, 2 and 3 in the respective columns. In the first row aggregate demand curves are shown for the beginning (blue solid line) and end period (pink dashed line) of a single simulation run. Each aggregate demand function is calculated by aggregating across our 22 agents their desired long and short positions for a given price. The price where aggregate demand is zero is the Walrasian equilibrium. This same run is again presented in the second row as a time series of the transaction prices (in red) that were solved for by the CDA and the bilateral exchange among traders. The starting prices in the CDA algorithm was initially set for the bid and ask at  $p_b=120$  and  $p_a=130$ . The blue line is the *ex post* solution for the Walrasian equilibrium for every simulation step, solved with *Mathematica's* fixedpoint algorithm: `FindRoot`; which uses Newton's Method. This equilibrium price solution does not have the bid-ask bounce that the CDA has, but as seen in market 2, it has many more problems solving for the stable equilibrium price when multiple equilibria exist than the CDA algorithm. The Walrasian equilibrium price path(s) evolves over time due to changes in wealth of our agents in the CDA trading simulation. When the Walrasian equilibrium makes a sudden jump it is almost always due to the presence of multiple equilibrium, and the jumping from one equilibrium to the other.

Using the *Mathematica* solution as a benchmark, we can consider the convergence of price to its equilibrium path for different order-flows or trade sequencing. The average of 7 runs (black solid line) and  $\pm 1$  standard deviation across the 7 different runs (pink dashed line) is presented in the third row of Table 4. The average Walrasian equilibrium is the flatter blue line in each of these diagrams in row 3.

The CDA algorithm of random ZP traders does not immediately jump to the equilibrium but in all simulations converges towards the stable equilibrium point even when there were potentially destabilizing equilibrium points. In market 1 the uniform distribution of reservation prices for the ZP speculators led to a unique Walrasian equilibrium which was effectively stable over the period. In market 2 where reservation prices were binomial, the market typically had 2 stable equilibrium points either side of the unstable equilibrium in market 2 when the net hedge was zero, and in market 3, when the net hedge was equal to 5000, it often had a single stable and single unstable fixed point. Despite the multiple equilibriums, in both cases the CDA converged to the stable point. It took longer to converge in market 3 with the 5000 net hedge given the limitation on trade size for the hedgers. While the price series was initially dominated by the speculators' expectations, the much higher fundamental demand kept the higher equilibrium price as a constant gravitation point for the CDA prices. This shows that the Cliff and Bruten critique (1997) that prices in ZI models were biased by the initial distribution of speculator reservation prices rather, than the Walrasian equilibrium, is inaccurate. While the distribution of speculator reservation prices do impact the evolution of the price series, long run gravitation is still to the Walrasian equilibrium even with random trading.

The faster converge appears under the binomial distribution or reservation prices rather than the meandering of the uniform distribution of traders. The more dispersed trader reservation prices the faster is convergence, but once speculators have used up their purchasing power and are constrained by their budgets then the uniform distribution of reservation prices for the ZP traders leads to a smaller bid ask spread and smaller price jumps in a liquidity crisis, for a market without limit orders.

The sequencing of ZP speculator entry into the bidding process makes quite a big difference to the different price paths as shown by the large standard deviation across runs in row 3. As a reminder, with no limit order book, it is the speculator's demand for immediacy and the random order flow which replicates the random trading emphasized in the standard ZI model where traders patiently add their random quotes to a limit order book.

On top of convergence, we can also consider the allocative efficiency of these futures markets. Drawing from Smith (1962)<sup>15</sup> we take our  $t=500$  transaction prices  $p_j : j = 1 \dots t$ , and formulate a price convergence measure of  $\alpha$  to  $P_j$  which is the theoretical equilibrium price over time  $t$ :

$$\alpha = \frac{100\sigma}{\frac{1}{t} \sum_{j=1}^t P_j} \quad \text{where} \quad \sigma = \sqrt{\frac{1}{t} \sum_{j=1}^t (p_j - P_j)^2}$$

We calculate  $\alpha$  for each individual run, then average this across our 7 simulations. The closer  $\alpha$  is to zero the closer the CDA price series is to the price that equates supply and demand for each point on average over the period  $t=500$ . Gode and Sunder (1997) found that their stock market with no wealth dynamics reached 1, but despite our much more volatile market structure with short selling and leverage, our model still produces a relatively low level of inefficiency. This average root mean squared difference between the CDA actual and equilibrium prices might also be thought of as a measure of price resiliency in the market.

	Market 1: U	Market 2: Bi	Market: 3 Bi, NH5000
$\alpha$	5.56	6.44	6.89

**TABLE 3** Average root mean squared difference between CDA actual and equilibrium prices

The price volatility that remains in these markets, after convergence, is due to the manner of bilateral trade and can be described as market liquidity constraints. This is distinctly different from the price volatility that comes from exogenous information shocks or adaptive expectations which are omitted from this study. Liquidity constraints such as wealth constraints, frequency of

<sup>15</sup> Smith (1962) created a price convergence measure,  $\alpha = 100 \sigma_0 / P_0$  where  $P_0$  is a static theoretical equilibrium price given by the intersection of the aggregate supply and demand curves, and  $\sigma_0 = \sqrt{\frac{1}{t} \sum_{j=1}^t (p_j - P_0)^2}$ . Since the model here has a dynamic concept of wealth the theoretical Walrasian equilibrium is also evolving over time.

margin trades, order-flow, trade capacity, and transaction costs all produce considerable price and quantity feedbacks, especially in a leveraged market with short selling. It is thought that this model produces a fertile ground for further study in this area. For example, when speculators are on their budget constraints, they will liquidate some of their position when prices move against them in order to stay within their margin requirements. This creates backward-bending demand functions, and multiple equilibria as introduced in Section 3.2 and can lead to spikes or large shifts in the equilibrium price series. In previous work with models of adaptive expectations, Ussher (2005) found that a low or very high margin requirement is more likely to reduce price volatility than a middle range one. Transaction taxes increased the standard deviation of prices but reduced kurtosis Ussher (2004). In settlement policies, Farmer et al. (2004) found that a higher settlement frequency can have a positive impact on price volatility. In our model, at a representative agent level it might appear prudent to require a high-risk speculative trader to mark-to-market to reduce default risk, but it is this RTGS that promotes the backward bending demand function. These types of liquidity risks are especially relevant to futures markets since counterparty risk is removed by the exchange by backing every trade, and regulations on settlement, margins, and transaction costs are easily implemented in the members only, exchange environment.

## CONCLUSIONS

In this paper we create a futures market model, with no limit order book (greater than 1), and impatient agents who offer their exogenous reservation (worst) market orders as soon as they arrive in the market, from a random trading sequence. Wealth follows a dynamic process impacted by market microstructure. Market orders that land between the bid-ask spread will replace the current bid or ask prices. It is argued that this model offers an extension to the zero-intelligence stock market methodology of Gode and Sunder (1993) where patient traders post random limit orders, constrained by their exogenous reservation price, to a limit-order book. In both models, heterogeneous traders are endowed with reservation prices determined by some random distribution. Random trading in the ZI market is in the posting of limit orders, mediated through a CDA best bid-ask quotes. Random trading in our futures model is through the sequencing ZP traders to place market orders mediated through the CDA best bid-ask quotes. It

is argued that in both cases, the CDA is the central algorithm by which the market clears and prices converge to the long run equilibrium without the necessity of agents optimizing their expected capital gains in some manner. Both markets lead to the convergence of prices to the equilibrium without a Walrasian auctioneer and without speculator optimization. The only meaningful difference between random limit-orders or randomly sequenced reservation market-orders, is in the impact on price volatility due to market liquidity. Both ZI and ZP markets can give insights into liquidity and its impact on price volatility, within the CDA of stock or futures markets, respectively.

This paper negated the Cliff and Bruten (1997) critique of ZI models, showing that when endowment is directly incorporated into the model, prices do not have to converge to expectations based on the probabilities of limit orders or the distribution of reservation prices. Even when adding a non-zero sum to a symmetric distribution of speculator reservation prices separating the Walrasian equilibrium from that of speculator expectations, the market still converged to the Walrasian equilibrium. This is because the final arbiter in ZI and ZP markets is the current bid-ask price, which will work to clear the market over time, when agents are wealth constrained.

The main finding of our model is that even with the removal of a limit order book, the addition of leveraged trading, short selling and transactions costs, the CDA will still brought about convergence to the equilibrium price in a small population of random but risk-neutral speculative traders. This confirms that original insight by Gode and Sunder (1993 & 1997): the imposition of scarcity, not intelligent optimization, is all that is required for allocative efficiency.

## APPENDIX I

The risk-neutral speculator maximizes next periods expected wealth (1). The first four boundary constraints represents the limit on a speculator's investment by the margin requirement when one is short in futures, (2) and (3), versus the extent to which futures can be bought long, (4) and (5). We have two each of these restrictions to take into account the one-way tax on both buys and sells  $\varpi p_t |x_t - x_{t-1}|$  for speculator  $i$ . If the transaction tax is positive then this boundary constraint will be slack. This dual tax restriction also impacts the budget constraint, (6) and (7). The bankruptcy conditions, (8) through (10), stop money wealth from going below zero.

For ZP speculator  $i$ :

*Maximize:*

$$\pi_{t+1}^e = (p^\theta - p_t)x_t + m_t \quad (1)$$

*Subject to:*

$$p_t x_t \geq -\kappa \left( (p_t^m - p_{t-1}^m)x_{t-1} + m_{t-1} - \varpi p_t (x_t - x_{t-1}) \right) \quad (2)$$

$$p_t x_t \geq -\kappa \left( (p_t^m - p_{t-1}^m)x_{t-1} + m_{t-1} + \varpi p_t (x_t - x_{t-1}) \right) \quad (3)$$

$$p_t x_t \geq \kappa \left( (p_t^m - p_{t-1}^m)x_{t-1} + m_{t-1} - \varpi p_t (x_t - x_{t-1}) \right) \quad (4)$$

$$p_t x_t \geq \kappa \left( (p_t^m - p_{t-1}^m)x_{t-1} + m_{t-1} + \varpi p_t (x_t - x_{t-1}) \right) \quad (5)$$

$$m_t \leq (p_t^m - p_{t-1}^m)x_{t-1} + m_{t-1} - \varpi p_t (x_t - x_{t-1}) \quad (6)$$

$$m_t \leq (p_t^m - p_{t-1}^m)x_{t-1} + m_{t-1} + \varpi p_t (x_t - x_{t-1}) \quad (7)$$

$$0 \leq (p_t^m - p_{t-1}^m)x_{t-1} + m_{t-1} - \varpi p_t (x_t - x_{t-1}) \quad (8)$$

$$0 \leq (p_t^m - p_{t-1}^m)x_{t-1} + (p_t^m - p_t)(x_t - x_{t-1}) + m_{t-1} + \varpi p_t (x_t - x_{t-1}) \quad (9)$$

$$m_t \geq 0 \quad (10)$$

## APPENDIX II

*Ask* is the quoted price at which a trader is willing to sell a security for, also known as an *offer*.

*Bid* is the quoted price at which a trader is willing to buy a security.

*Bid-ask spread* is the difference between the best (highest) available *bid* and best (lowest) available *ask*.

*Derivative* contracts are instruments that derive their value from the values of the underlying instruments upon which they are based. Derivatives include forwards, futures, options and swaps.

*Immediacy provider* is any trader who has standing *bids* or *asks* (in the *limit order-book*) and thereby provides the ability for other traders to transact immediately if they wish.

*Initial margin* is the minimum amount of collateral specified by the exchange that has to be deposited in the trader's margin account at the exchange in order to open a derivative contract. It can be cash, or Treasury bills, kept in trader's margin account along with the open position. The margin is used by the exchange to ensure counterparty contract fulfillment.

*Limit order* is a standing quote indicating that various traders are ready to buy or sell a specified amount of a security at a specified price.

*Limit order-book* is a record of unexecuted limit orders maintained by a dealer or exchange. *limit-orders* stay on the book until rescinded.

*Liquidity* is often referred to as "depth, breadth and resiliency" - Depth and breadth mean that sufficient interest exists on both the sell side and the buy side for traders to be able to execute a large number of transactions in a short period of time. Resiliency means that there is a large "countervailing order flow whenever transaction prices change because of temporary order imbalances" (Garbade p. 428 cited in Bernstein 1987, p. 55). A thick or compact distribution of limit orders on the books is an indicator of a liquid market.

*Limit traders* provide liquidity and trade by quoting *bid* or *ask* prices rather than making certain transactions with *market orders*.

*Liquidity trader* is a trade that is not issued to capitalize on price change, but issued for an exogenous or unknown reason.

*Margin* is the collateral that is deposited in a margin account with the exchange. Trading on margin means that a trader is leveraged and has deposited only a proportion of the contract value with the exchange. For example, a 1% margin means that \$1,000 of funds is on deposit backing a \$100,000 contract position.

*Margin account* is a trading account held with the exchange, jointly owned by the trader and the exchange. All cash, t-bills and futures contracts are held in this account. In the futures market traders can earn interest on *margin* deposited in the account if held as T-bills. This margin is used as collateral to back any open futures position to guarantee contract fulfillment.

*Margin Call* is issued by the exchange/broker to a trader when the margin posted is less than the minimum or *maintenance margin* requirement. Should the balance in your margin account fall below the *maintenance margin*, the trader must either deposit funds or securities sufficient to restore the balance to the *initial margin* level, or liquidate the *open position*.

*Maintenance margin* is the minimum margin requirement and typically some fraction of the *initial margin* which the trader must maintain to avoid a *margin call*. If the maintenance margin is equivalent to the initial margin then the *margin call* is the *variation margin*.

*Market makers*, or dealers supply liquidity, quoting bid and offer prices at which they stand ready to buy and sell.

*Mark to market* is the adjustment of an account to reflect accrued profits and losses to current or market prices.

*Market order* is a request to buy at the quoted *ask* (lift the ask) or sell at the quoted *bid* (hit the bid). Market orders remove limit orders from the book and thereby remove liquidity from the market.

*Mid-price* is the average of the best bid and ask prices quoted in the market.

*Momentum traders* buy when prices are rising and sell when prices are falling, destabilizing prices. They often implement their strategies using *stop orders* to their brokers, or monitor the market themselves and issue standard *market orders*.

*Noise traders*, in an objective world, are those who think they are trading on information, but in reality are trading based on misinformation or noise. They may also be equated with *liquidity traders*.

*Offer*, see *ask*.

*Off-set* is the closing out or liquidation of an open market position.

*Open-outcry* also known as a continuous bilateral oral auction, where traders negotiate face to face and the market uses rules to match orders.

*Open Interest* is the total number of outstanding option or futures contracts that have not been closed out by offset or fulfilled by delivery.

*Order-driven market* is one where buyers and sellers regularly trade with each other without the intermediation of dealers. Order precedence rules determine who exchanges with who. In contrast, see *quote driven market*.

*Order-flow* is the sequence of trading

*Quote-driven market* is one where almost all trades are intermediated by dealers who quote a bid and ask at the same time.

*Real time gross settlement* is the continuous *settlement* of payments or *margin calls* in real time on a transaction by transaction basis e.g. interbank or central bank settlement.

*Scalpers* are traders who buy and sell for their own account. They try not to hold large positions for more than a few minutes. They are continuously acquiring and unwinding their position, holding a zero position on average.

*Spread* is the difference between the *bid* and *ask* quoted prices of a security.

*Spot price* is the current transaction price for exchanging a physical commodity or financial asset for cash, as opposed to a *futures price*.

*Short sale* is the sale of a security not owned by the seller at the time of the trade. Short sales are usually made in anticipation of a decline in the price.

*Settlement prices* are official prices calculated by the exchange to calculate *margin call* payments. Formulas vary and may depend upon how active trading is. Generally, settlement prices are calculated as some average of transaction prices depending on settlement frequency. For end of day settlement the settlement price is an average of prices immediately before the close. In real-time gross settlement the settlement price might be the *mid-point price*.

*Transaction price* is the actual trading price when a market order crosses a limit order (*bid* or *ask* quote).

*Variation margin* is the running profit or loss paid or received on a daily basis. If a trade is opened then the variation margin is calculated against the *settlement price* set by the exchange and paid according to the settlement frequency. For long positions, if the price has moved upwards this represents a profit, which will be received through a variation margin payment, credited to the *margin account*.

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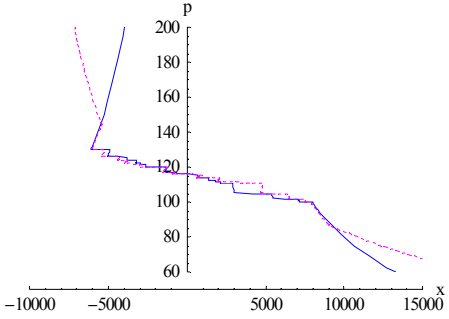
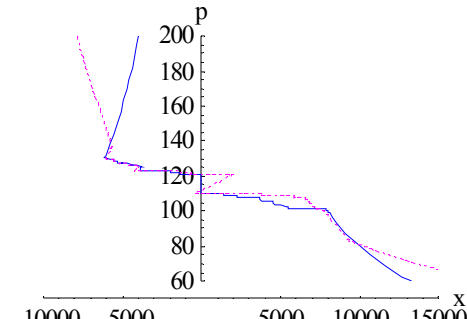
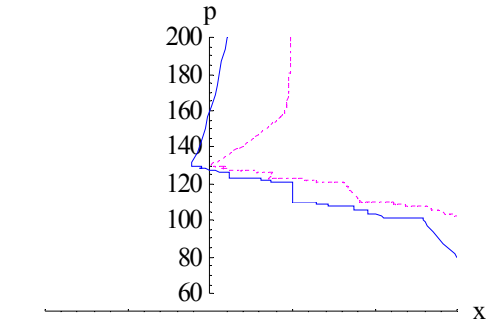
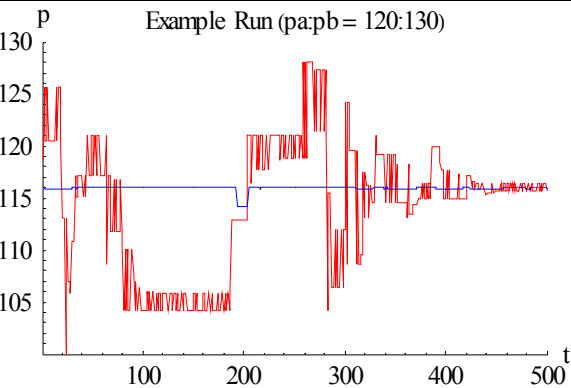
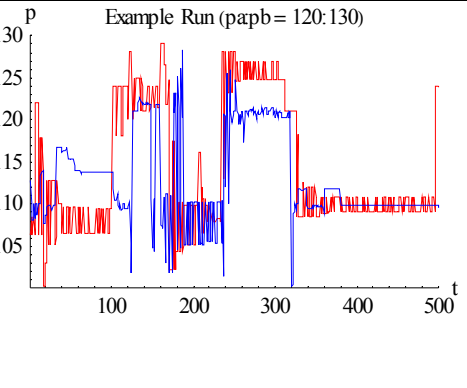
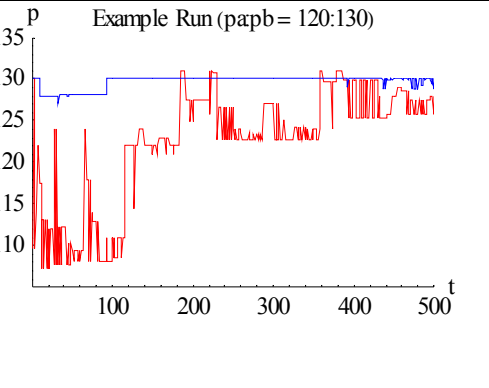
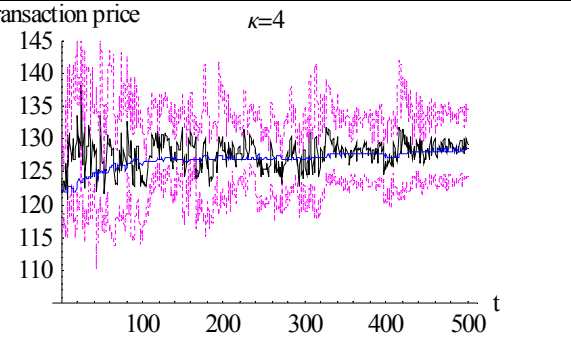
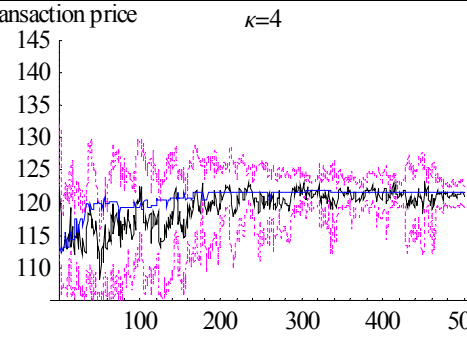
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Table 4:

<p>Uniform Distribution Price Expectations                  20 speculators pe from <math>U\sim(110,150)</math>                  Net hedger demand = 0</p>	<p>Binomial distribution Price Expectations                  10 speculators pe from <math>U\sim(100,110)</math>                  10 speculators pe from <math>U\sim(120,130)</math>                  Net hedger demand = 0</p>	<p>Binomial distribution Price Expectations                  10 speculators pe from <math>U\sim(100,110)</math>                  10 speculators pe from <math>U\sim(120,130)</math>                  Net hedger demand = 5000</p>
		
<p>Example Run (pa:pb = 120:130)</p> 	<p>Example Run (pa:pb = 120:130)</p> 	<p>Example Run (pa:pb = 120:130)</p> 
<p>transaction price <math>\kappa=4</math></p> 	<p>transaction price <math>\kappa=4</math></p> 	<p>transaction price <math>\kappa=4</math></p> 