Capital Gains and Housing as a Quasi-Giffen Good

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Abstract

This paper studies the effect of potential capital gains in owner-occupied housing on the individual consumer’s demand for housing. In the theoretical part of the paper, we use a two-period life cycle model with uncertainty to derive testable predictions that the own-demand functions for the standard consumer goods have the traditional downward sloping property while allowing for the possibility that the own-demand function for housing, a non-inferior good, may be upward sloping. This makes for a clear theoretical distinction between housing and the other consumer goods. In the empirical part of the paper, we estimate tenure choice and housing demand equations for PUMA samples from two large metropolitan areas, San Francisco and Atlanta, and find contrasting results: housing demand is downward sloping in the former but upward sloping in the latter. Our theoretical model reconciles these conflicting results and provides a rationale for each of them.

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Housing’s dual roles of consumption good and investment asset have made it an object of special interest and study. For example, Brueckner (1997) analyzed how these components affect consumers’ asset portfolio choices and Piazzesi et al. (2007) incorporated them in a consumption-based asset pricing model. Housing’s dual roles also have intriguing implications for the fundamental properties of housing demand functions. If housing were to be a pure, non-inferior consumption good, then the individual consumer’s own demand curve for it would be unambiguously downward sloping. However, housing is also an investment asset, with potential for capital gain or loss. During periods of rising housing prices, an increase in the price of housing can be taken as a signal for expectation of capital gain and housing demand could increase. Similarly, during periods of declining housing prices, a price decrease can signal expectation of capital loss and demand could decrease. Should this asset component response of housing demand dominate the traditional neoclassical demand response, the own-demand curve would be upward sloping. In this case, housing portrays the role of a quasi-Giffen good, i.e., a good whose own-demand slopes upward even though it is non-inferior.\footnote{Other examples of goods that can sometimes be quasi-Giffen are restaurant eating [Becker (1991)] and various conspicuous consumption goods such as fashionable clothes, high prestige cars and jewels [Veblen (1934), Corneo and Jeanne (1997)]. The latter are goods which are purchased because of the demonstration effects that their consumption exerts on others.}

In this paper, building on the work of Dusansky and Wilson (1993) and Dusansky and Koç (2007), we present a two-period model of consumption good and housing services demand in which the dual roles of housing are captured and for which the empirical properties of housing demand are distinctly different from those of the standard consumption goods. In this model it is possible for housing to be quasi-Giffen \textit{at the same time} that the non-housing good demand functions exhibit the traditional properties. This combined result is new to the literature on housing demand. We also undertake econometric testing that goes
beyond examination of the anecdotally reported "hot" housing markets that were analyzed in Dusansky and Koç (2007). Two large and geographically distinct metropolitan locations are studied, Atlanta and San Francisco, employing data for forty nine Public Use Metropolitan Areas (PUMAs). We find strong econometric support for a downward sloping housing demand curve in one location and for an upward sloping demand curve in the other. Special cases of our model provide a theoretical rationale for each of these findings.

1. The Model

1.1. Description of Choices. The theoretical framework embodies a two-period life cycle model of consumption goods and housing services demand under intertemporal uncertainty. A central role is given to the formulation of expectations about uncertain future prices; the links between the current housing price and expectations of future prices; the consequences for perceptions of potential capital gains; and the implications for the effect of changes in the current housing price on owner-occupied housing demand. The economy consists of pre-retirement workers and non-working retirees and our representative consumer plans for two periods. In period 1, he is taken to be a homeowner, purchasing consumption goods and owner-occupied housing. In period 2, our consumer is assumed to be in retirement and to be a renter, choosing consumption goods and rental housing. Owner-occupied housing is sold at the beginning of this period and the sales proceeds are devoted to consumption and rental housing. The utility function, suitably well-behaved,\(^2\) is given by

\[
U = U(x_1, h^o_1, x_2, h^r_2)
\]

\(^2\)The consumer’s utility function is assumed continuous, bounded, concave and order preserving, as well as appropriately differentiable.
where \( x_1 = (x_{11}, \ldots, x_{n1}) \) and \( x_2 = (x_{12}, \ldots, x_{n2}) \) represent the vectors of regular consumption goods in the respective periods, \( h_1^o \) depicts units of owner-occupied housing services in period 1 and \( h_2^r \) represents units of rental housing services in period 2.\(^3\)

The first period budget constraint is given by

\[
(2) \quad \sum_{i=1}^{n} p_{i1}x_{i1} + p_{o1}h_1^o = Y
\]

where the \( p_{i1} \) represent the prices of the consumption goods, \( p_{o1} \) depicts the price of owner-occupied housing and \( Y \) is certain earned income or wealth. Housing stock is carried into the subsequent retirement period. Owner occupied housing thus serves both as a consumption good and as an investment asset for intertemporal transfer of wealth.\(^4\)

The second period budget constraint is given by

\[
(3) \quad \sum_{i=1}^{n} p_{i2}x_{i2} + p_{r2}h_2^r = S + p_{o2}h_1^o
\]

where \( p_{r2} \) depicts the second period price of a standardized unit of rental housing services and \( S \) is fixed exogenous income (e.g., Social Security income payments). Now the consumer is in his retirement years. He consumes rental housing and consumption goods, which are financed by \( S \) augmented by the proceeds of the sale of the housing stock.

The consumer faces intertemporal uncertainty; current prices are known, but future prices are unknown. Expectations about these uncertain second period prices are formulated and taken to be based on the known prices prevailing in the first period. These individual

\(^3\)The underlying general equilibrium framework specifies a two-period planning horizon in an economy with an infinite number of finite periods. One can prove the existence of a general competitive equilibrium for which all prices, including the rental price and the price of owner-occupied housing, are endogenously determined. See Dusansky and Wilson (1993) for details.

\(^4\)Consumer savings can easily be incorporated into the model, so that both savings and housing as a capital asset can be carried into the subsequent period. One can also introduce other assets (e.g., money, bonds). None of these extensions would change the qualitative results presented below, but they would add to the complexity of the model. See Brueckner (1997) for a treatment of housing as part of the portfolio choices of homeowners. He shows that housing’s dual roles can lead to an inefficient over-investment in housing; its reduction would involve a consumption loss.
expectations are depicted by the conditional subjective probability distribution $F(p_2|p_1)$, where $p_i = (p_{1i}, \ldots, p_{ni}, p_{oi}, p_{ri})$ denotes the price system that prevails in period $i, i = 1, 2$.

1.2. Demand Behavior. In the light of the consumer’s utility function and expectation formulations, expected utility is defined as

$$V_1(x_1, h_{1i}^0, p_1) = \int_{p_2} U(\phi(x_1, h_{1i}^0, .)) dF(.|p_1)^6$$

where $V_1(.)$ is the von Neumann-Morgenstern expected utility function of the action $(x_1, h_{1i}^0)$ if $p_1$ is quoted in period 1. It is assumed that the consumer maximizes expected utility, so that the optimization problem becomes maximizing (4) subject to (2). In the usual manner, the first order conditions can be solved for the consumption good demand functions and the demand function for owner-occupied housing, and the properties of the demand functions can be determined from the respective Slutsky equations that are deriveable from a total perturbation of the first order conditions. Letting $q = (x, h^c)$, we have

$$\frac{\partial q_i}{\partial p_k} = -q_k \frac{\partial q_i}{\partial Y} + \lambda \frac{D_{ki}}{D} - \left[ \sum_{j=1}^{n} V_{q_jp_k}(.) \frac{D_{ji}}{D} + V_{h^c p_k}(.) \frac{D_{n+1;i}}{D} \right], \forall i, k$$

where $V_{q_j p_k}(.) = \frac{\partial^2 V(.)}{\partial q_j \partial p_k}$, $D$ is the determinant of the Jacobian of the first order conditions, $D_{ji}$ is the cofactor of the element in row $j$, column $i$, $i, j, k = 1, \ldots, n$ refer to $n$ consumer goods.

5 Formally, $F: p_1 \rightarrow \mathcal{M}(p_2)$, where $\mathcal{M}(p_2)$ is the set of probability measures defined on the measurable space $(p_2, \mathcal{B}(p_2))$ and where $\mathcal{B}(p_2)$ is the Borel sigma algebra of $p_2$, i.e., the sigma algebra generated by its open subsets. By introducing the usual topology of weak convergence of probability measures on the set $\mathcal{M}(p_2)$ and assuming that the expectations mapping $F$ is continuous, we can represent the consumer’s expectations by $F(p_2|p_1)$, which is the subjective probability distribution for period 2’s prices, given period 1 prices.

6 We define $\phi(\bar{\pi}_1, \bar{h}_{1i}, p_2)$ as the set of conditional consumption programs associated with the price system $p_2$, which is a subset of the feasible consumption bundles for the two periods such that $x_1 = \bar{x}_1$ and $h_{1i}^0 = \bar{h}_{1i}$, assuming that the second period budget constraint is satisfied and utility is maximized. Since the ultimate consequences af any action $(x_1, h_{1i}^0)$ in period 1 are the individual’s consumption bundles for the two periods, the set of elements in $\phi$ consist of $(x_1, h_{1i}^0, x_2, h_{1i}^0)$.

7 Since $U(\phi(.))$ is a well-defined continuous function and is bounded, $V(.)$ is well-defined. By construction, it inherits the usual regularity properties (i.e., continuity, concavity, “more is better,” etc.) and satisfies the axioms of expected utility.

8 The optimal solution values for $x_2$ and $h_{1i}^0$ in the second period follow directly.

9 Details of the derivation are available upon request.
and $i, j, k = n + 1$ refer to owner-occupied housing services, $\lambda$ is the Lagrange multiplier in the maximization of (4) subject to (2). For convenience, we have dropped the time subscript with the understanding that all variables are for period 1.

The Slutsky equations for the own-demands are given by

$$
\frac{\partial q_i}{\partial p_i} = -q_i \frac{\partial q_i}{\partial Y} + \lambda \frac{D_{ii}}{D} - \left[ \sum_{j=1}^{n} V_{q/p_i}(\cdot) \frac{D_{ji}}{D} + V_{h^p_i}(\cdot) \frac{D_{n+1,i}}{D} \right], \forall i = 1, ..., n + 1
$$

where $i = 1, ..., n$ refers to $n$ consumer goods and $i = n + 1$ refers to owner-occupied housing services. The left-hand side of (6) is the uncompensated (i.e., the Marshalian) price effect. The first and the second terms on the right-hand side of (6) consist of the weighted income effect and the substitution effect, respectively, common to neoclassical theory. They represent the effect of a change in own-price on the optimal value of own-demand through the first period budget constraint. The term in squared brackets on the right-hand side of (6) is the effect of a change in own-price on the optimal value of own-demand by altering the objective function (i.e., the expected utility function in (4)), which we designate as the “expectation effect.”

The expected utility function is affected by own prices because it includes $p_1$ as an argument. The expectation effect arises because the individual forecasts second period prices on the basis of first period prices and because the second period prices determine the optimal consequences of any first period action $(\bar{x}_1, \bar{h}_1)$ and hence the utility of that action.

The utility of a current period action changes in response to price changes insofar as the permissible actions in the second period change.

It is important to note that the Slutsky equations for all the goods, including housing, are not the same as they appear in traditional microeconomics. The right hand side of the Slutsky equations for the own-demand contain the usual terms $-q_i \frac{\partial q_i}{\partial Y} + \lambda \frac{D_{ii}}{D}$ and additional terms. These additional terms reflect the effect of a changing own price on the expected utility function, operating through the expected marginal utilities of all consumption goods.
and housing. As a result, the sign of the right hand side of the own-demand Slutsky equations is ambiguous. Non-inferiority is not sufficient to ensure the traditional result; we do not have the prediction that the own-demands for any of the goods is downward sloping. In fact, it is entirely possible for these own-demand curves to be upward sloping.\textsuperscript{10}

1.3. **Distinguishing between Housing and the Standard Consumer Goods.** We now modify the model for the purpose of establishing a clear distinction between housing demand and that of the standard consumer goods. We seek to reinstate the traditional demand properties for the standard goods while preserving the possibility that the housing own-demand curve may be upward sloping.\textsuperscript{11} The key modifications embody the assumptions that the underlying utility function is intertemporally separable, a restriction common to many applied fields, and that the consumer employs a general forecasting scheme with additive error term.\textsuperscript{12} Starting with the von Neumann-Morgenstern expected utility function in (4), recalling that
\[
x_2 = x_2(h_1^o, p_2),
\]
\[
h_2 = h_2(h_1^o, p_2),
\]
and introducing the two simplifications, the expected utility function becomes
\[
U^1(x_1, h_1^o) + \int_{\epsilon_1} W(h_1^o, g(p_1) + \epsilon_1) f(\epsilon_1) d\epsilon_1,
\]
where $\epsilon_1$ is the random variable. We next use Taylor’s theorem and expand $W(\cdot)$ around $g(p_1)$ regarding $p_2$ as a unique random variable while treating $h_1^o$ as exogenous and substitute the resulting expression in (7). This yields
\[
U^1(x_1, h_1^o) + W(h_1^o, g(p_1)) + \sum_{i=1}^{n-1} \frac{W^i(h_1^o, g(p_1))}{i!} M_{\epsilon_1}^i + \frac{W^n(h_1^o, \delta_1)}{n!} M_{\epsilon_1}^n,
\]
\textsuperscript{10}In this case, consumption goods and housing are not Giffen goods because they are non-inferior.
\textsuperscript{11}Here we employ the methodology of Dusansky and Wilson (1993). Their special case recovered the traditional demand properties for all the goods, consumption and housing alike, leaving no distinction between them for empirical testing.
\textsuperscript{12}This is depicted by
\[
p_2 = g(p_1) + \epsilon_1,
\]
where $g(p_1) = E[p_2|p_1]$, $\epsilon_1$ are identically, independently distributed with zero mean and constant variance, which is a general way of delineating how the consumer predicts future prices based on current ones.
where \( \int_{\epsilon_1} \epsilon_1 f(\epsilon_1) d\epsilon_1 = M_{\epsilon_1}^i \) is the \( i \)th moment of the random variable \( \epsilon_1 \). Noting that \( \delta_1 \) is some constant between \( p_2 \) and \( g(p_1) \) and that all moments are some constants, we have

\[
U^1(x_1, h^\circ_1) + Z(h^\circ_1, p_1) + \Gamma(h^\circ_1).
\]

We now allow for a partially separable real wealth effect, which additively separates consumer good prices from the middle term, to arrive at

\[
U^1(x_1, h^\circ_1) + g(h^\circ_1, p_{o1}, p_{e1}) + h(p_{e1}) + \Gamma(h^\circ_1). \tag{10}
\]

We know that this expected utility function achieves our goal because it is a special case of a more general class of functions that does so (see the appendix for a formal derivation of this class).

As a consequence of the specification in (10), the Slutsky expressions for the consumption good own-demand curves are

\[
\frac{\partial x_i}{\partial p_i} = -x_i \frac{\partial x_i}{\partial Y} + \lambda \frac{D_{i,i}}{D}, \forall i = 1, ..., n. \tag{11}
\]

The housing own-demand curve is given by

\[
\frac{\partial h^\circ}{\partial p_o} = -h_o \frac{\partial h^\circ}{\partial Y} + \lambda \frac{D_{n+1,n+1}}{D} - V_{h^\circ p^\circ o} \frac{D_{n+1,n+1}}{D}. \tag{12}
\]

Thus, the traditional own-demand properties are reinstated for the standard consumer goods while the sign of \( \frac{\partial h^\circ}{\partial p_o} \) remain ambiguous. It is possible for it to be positive.

This result on housing demand highlights the importance of modeling the links between the current housing price and expectations of future prices, and the consequences for perceptions of capital gains. The model and its comparative statics properties allow for the possibility that an increase in the price of owner-occupied housing can lead to the expectation that next period’s housing price will be higher, resulting in a capital gain on the housing stock. This expectation of capital gain can increase the quantity of owner-occupied housing services
demanded. The observed result could be that an increase in housing price leads to an increase in owner-occupied housing demand.

It is useful to establish conditions under which an upward-sloping housing demand curve would be an unambiguous prediction of the model. A sufficient condition for this prediction is that the expectation effect (the third-term on the right-hand side of (12)) outweighs the sum of the income and substitution effects (the first two terms on the right-hand side of (12)). But since the income and substitution effects are negative, a necessary condition for an upward-sloping housing demand curve is that the expectation effect is positive. Since we know that $\frac{D_{n+1,n+1}}{D} < 0$ by the second order conditions, the necessary condition is satisfied when $\frac{\partial^2 V(\cdot)}{\partial h \partial p_o} = V_{h_o p_o} > 0$, i.e., the marginal expected utility of owner-occupied housing is increasing with respect to its own price.

We now present new econometrics results which show, depending on the housing market, that it is possible to observe both upward-sloping and downward-sloping own-demand curves for owner-occupied housing services.

2. THE DATA AND THE ECONOMETRIC MODEL

2.1. Sample Selection. We study samples from the 1990 Survey of Population and Housing. The universe description for the original data is all persons and housing units in the United States. We use a version of the data set which contain records representing 5 percent samples of the housing units in the United States and the persons in them. The geographic coverage of the data set identifies various subdivisions of states called Public Use Microdata Areas (PUMAs), each with at least 100,000 persons. These PUMAs are primarily based on counties, and may be part of a county, whole counties or groups of counties.
We choose two major urban housing markets for our empirical analysis: San Francisco and Atlanta. These represent two distinctly different geographical locations in the US; Atlanta is in the Mid-Atlantic region and San Francisco is in the West Coast. To define the San Francisco housing market, we use the metropolitan San Francisco area which consists of Central San Francisco and four counties surrounding the city. We therefore pick 30 PUMAs covering the following sub-areas: Central San Francisco, Marin County, Contra Costa County, Alameda County and San Mateo County. To define the Atlanta housing market, we use the 10-county metropolitan area defined by the Atlanta Regional Commission. We pick 19 PUMAs covering the following 10 counties: Fulton, Cobb, Gwinnett, Dekalb, Douglas, Fayette, Cherokee, Clayton, Henry and Rockdale. The list of counties and PUMAs covering the San Francisco and Atlanta housing markets is provided in Table 1.
Table 1. The List of Counties and PUMAs Covering the San Francisco and Atlanta Housing Markets

<table>
<thead>
<tr>
<th>San Francisco County</th>
<th>PUMA</th>
<th>County</th>
<th>PUMA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marin</td>
<td>01501</td>
<td>Cherokee</td>
<td>00600</td>
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<tr>
<td></td>
<td>01502</td>
<td>Douglas and Fayette</td>
<td>01500</td>
</tr>
<tr>
<td>Contra Costa</td>
<td>01700</td>
<td>Henry and Rockdale</td>
<td>01600</td>
</tr>
<tr>
<td></td>
<td>01801</td>
<td>Clayton</td>
<td>01700</td>
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<tr>
<td></td>
<td>01802</td>
<td>Cobb</td>
<td>01801</td>
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<td>01804</td>
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<td>01803</td>
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<tr>
<td></td>
<td>01805</td>
<td>Dekalb</td>
<td>01901</td>
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<tr>
<td>San Francisco</td>
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<td></td>
<td>01902</td>
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<td>01904</td>
<td>Fulton</td>
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<td>02002</td>
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<td>02003</td>
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<td>Alameda</td>
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<td>Gwinnett</td>
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<tr>
<td>San Mateo</td>
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<td>02206</td>
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</table>

There are two record types in the data set: the first includes a collection of housing records and the second includes a collection of person records. Each housing unit record is followed by a variable number of persons record, one for each occupant. However, we only use the person record information for the householder. Thus, while our geographic unit of analysis is a PUMA, our individual unit of analysis is the householder. We removed vacant units, group
quarters institutions, housing units rented for no cash, housing units on ten acres or more and housing units that are part of a condominium. Since our theoretical model assumes that retirees are not homeowners, we also removed those housing units for which the household head is 65 years of age and older, so that our data sample is consistent with the theory.

2.2. Econometric Model. Our analysis seeks to determine how household demand for housing services is affected by changes in current housing prices. In an attempt to quantify this relationship, we employ an econometric model of housing demand that considers the joint determination of tenure choice and quantity of housing services demanded. Following Rosen (1979), the empirical formulation of this joint determination is achieved by using the Heckman (1979) sample selection model.

2.3. Construction of Key Variables. The quantity of housing services is considered to be a function of personal characteristics of household head, household background and the price of housing. The personal characteristics of household head - such as age, race, sex, education, marital status, immigration status and disability status - are important since they capture the differences in tastes and expenditure patterns. Examples of household background variables are household permanent income, household transitory income, household dividend and interest income, household size and moving information. Household dividend and interest income captures whether the household is constrained in its housing demand by an inability to generate the required down payment. Following Turner (2003), a variable is created that takes the value 1 if the household has at least $400 in annual interest, dividend and rental income. Household size is defined as children plus adults living in the household. Moving information of whether or not the household moved in the last 5 years captures the different search costs and different demographic factors of mobile households. The definitions of these variables are reported in Table 2. Measurement of some of these variables requires
These are permanent income, transitory income, price of housing services and quantity of housing services.

Table 2. Definitions of Explanatory Variables Used in Regression Equations

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>Sex</td>
<td>1 if female-headed household</td>
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<tr>
<td>Age</td>
<td>Age of household head</td>
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<tr>
<td>White</td>
<td>1 if white</td>
</tr>
<tr>
<td>Mar</td>
<td>1 if married</td>
</tr>
<tr>
<td>Hhsize</td>
<td>Household size</td>
</tr>
<tr>
<td>Imm</td>
<td>1 if not a citizen of the U.S.</td>
</tr>
<tr>
<td>Edu</td>
<td>1 if above high school educational attainment</td>
</tr>
<tr>
<td>Inc</td>
<td>Current household income</td>
</tr>
<tr>
<td>Dis</td>
<td>1 if limited in kind or amount of work</td>
</tr>
<tr>
<td>Nmov</td>
<td>1 if the household did not move in the last 5 years</td>
</tr>
<tr>
<td>Perm Inc</td>
<td>Permanent income</td>
</tr>
<tr>
<td>Trans Inc</td>
<td>Transitory income</td>
</tr>
<tr>
<td>Div Inc</td>
<td>1 if annual interest, dividend and rental income is in excess of $400</td>
</tr>
<tr>
<td>Our Price</td>
<td>Owner occupied housing price</td>
</tr>
<tr>
<td>Rent Price</td>
<td>Rental housing price</td>
</tr>
</tbody>
</table>

Our data set does not contain an estimate of permanent income. Therefore, it is necessary to find a proxy for this variable. We follow Gillingham and Hagemann (1983), Rapaport (1997) and Ioannides and Zabel (2003) and use a regression approach to measure permanent income, which represents potential lifetime earnings. Since there is no direct observation on permanent income, the regression approach uses an instrumental variable approach to deal with this problem. The observed income is defined as the sum of permanent income and transitory income. If permanent income is a function of human capital assets (such as education and age) and nonhuman assets, then the regression of observed income on these variables provides permanent income as the fitted value of the regression and transitory income as the residual. To run the hedonic regression for permanent income, we use age, sex, race, education, marital status, unemployment status, immigration status, number of workers
in the household, whether the household resides in an urban or rural area, occupation, and information on whether the household head is employed in the private sector, public sector or is self-employed as personal characteristics variables. Permanent income is estimated using the equation

$$Y_p^{(\xi)} = \frac{Y_p^{\xi} - 1}{\xi} = \alpha_p + X_p'\beta_p + D\gamma_p + \varepsilon_p,$$

where $Y_p$ equals the observed income of the household, $X_p$ includes the personal characteristics determining permanent income, $D$ is a vector of PUMA dummies and $\xi$ is the Box-Cox nonlinear transformation parameter.\textsuperscript{13}

Neither the price of a standardized unit of housing services nor the number of standardized units consumed are available in our data set. We follow Rapaport (1997), Ioannides and Zabel (2003) and Quigley and Raphael (2005) and employ a hedonic regression approach to estimate the price of housing services. Self-reported house value\textsuperscript{14} is equal to the per unit price times the number of units consumed. Therefore, it is necessary to extract both price and quantity of housing services from this variable. It is assumed that the market value of owner-occupied housing is a function of structural characteristics and location specific characteristics. A reduced form hedonic regression for owners is run using a Box-Cox specification

$$\text{value}^{(\theta)} = \frac{\text{value}^{\theta} - 1}{\theta} = \alpha_v + X_v'\beta_v + D\gamma_v + \varepsilon_v,$$

where $D$ is a vector of PUMA dummies and $\theta$ is the Box-Cox nonlinear transformation parameter. $X_v$ includes information on number of rooms, number of bedrooms, plumbing facilities, kitchen facilities, heating fuel, means of sewage, age of the house, real estate taxes, premium for fire, hazard and flood insurance, owner costs, and whether the property is

\textsuperscript{13}Further details on the hedonic regression results including those for permanent income and housing prices (both for owners and renters) are available upon request.

\textsuperscript{14}This variable is householder's estimate of how much the property would sell if it were for sale. We transform discrete interval responses to a single value variable by choosing midpoints of the intervals.
located in an urban area. The second step in creating the owner-occupied housing price variable is to use the results of this hedonic regression to generate a fitted representation for the value of an identical housing unit in different PUMAs. Mean values of the variables in $X_v$ are calculated for the *whole* sample and then fitted values are generated for each PUMA. This yields market housing services price for each PUMA. Thus, fitted values represent the estimated price for the *same home* in different PUMAs, so that we are left with price comparisons that hold quantity constant. This procedure yields the price of a standardized unit of housing services. To find the number of standardized units, we divide the self-reported market value of the housing unit by the price of the standardized house for the PUMA containing the house. This yields the dependent variable for the housing demand regression equation.

The procedure outlined above provides variations in housing prices that would reflect changes in market conditions, since it creates housing prices for the same home across PUMAs. Once one controls for permanent income and all other household characteristics that affect housing demand, the estimated coefficient of this price variable from an owner-occupied housing demand regression could then be interpreted as follows: consider two identical households who already decided to own a home, one living in City $X$, the other living in City $Y$. Suppose that the household living in City $Y$ faces higher housing prices due to different market conditions. If this household’s housing demand is less than that of the household living in City $X$, then the price coefficient would be negative, i.e., the observed owner-occupied housing demand would be downward sloping. If, on the other hand, this household’s demand is higher, then the price coefficient would be positive, i.e., the observed owner-occupied housing demand would be upward sloping.
The dependent variable for the tenure decision model takes on the value 1 if a household owns its home and takes on the value 0 if a household rents it. This variable is considered to be a function of personal characteristics of the household head (such as age, race, sex, education, marital status, household size, immigration status, disability status), household background and the respective costs of owning and renting. Examples of household background variables are household permanent income, household transitory income, household dividend and permanent income, and moving information. Household permanent income is important since decisions about housing tenure are likely to be based on expectations regarding it. Household transitory income, on the other hand, is crucial in helping purchasers to overcome downpayment constraints. Moving information is useful because of the transactions costs involved in buying a home; the moving family is more likely to rent than to own. Similar to Ioannides and Rosenthal (1994) and Turner (2003), the costs of owning and renting are taken to be the estimated price of housing and the estimated price of renting, respectively.

We again use a hedonic regression approach to estimate the price of rental housing. The self-reported monthly rent paid by the household\(^\text{15}\) is regressed on the characteristics of the dwelling and a vector of PUMA dummies. We use the same housing characteristics variables that we use in the owner-occupied housing price hedonic regression except that we do not employ real estate taxes, insurance premium or monthly owner costs, but instead use a new variable, \(addf\), which is the sum of the key components of rent, which includes the costs of electricity, fuel, gas and water.

\(^{15}\text{We transform discrete interval responses to a single value variable by choosing midpoints of the intervals.}\)
3. Results

3.1. Tenure Choice Regression Results. The results of the tenure choice regression appear in Table 3. The statistically significant demographic variables in both markets are Age, Mar, Perm Inc, Trans Inc, Div Inc, Hhsiz, Nmov, Dis and Edu. Households in which the head of the household is older and married, whose head has a higher educational attainment, larger households, and households that move infrequently, with higher incomes and with higher interest incomes (i.e., less constrained in liquidity) have a higher probability of owning. On the other hand, households in which the head of the household is limited in amount of work have a lower probability of owning. White is significant in the Atlanta housing market. Households in which the head of the household is white have a higher probability of owning. Imm and Sex are significant in the San Francisco housing market and their coefficients indicate that households in which the head of the household is a non-US citizen have a lower probability of owning, whereas households in which the head of the household is female have a higher probability of owning. Lastly, our tenure choice regressions suggest that as owner-occupied housing prices increase, the probability of owning decreases, whereas as rental housing prices increase, the probability of owning increases.
Table 3. Tenure Choice Regressions

<table>
<thead>
<tr>
<th>Variable</th>
<th>San Francisco</th>
<th>Atlanta</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient</td>
<td>Marginal Effects</td>
</tr>
<tr>
<td>Sex</td>
<td>0.036**</td>
<td>0.014**</td>
</tr>
<tr>
<td></td>
<td>(2.15)</td>
<td></td>
</tr>
<tr>
<td>Age</td>
<td>0.028*</td>
<td>0.011*</td>
</tr>
<tr>
<td></td>
<td>(37.27)</td>
<td></td>
</tr>
<tr>
<td>White</td>
<td>0.010</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td>(0.62)</td>
<td></td>
</tr>
<tr>
<td>Mar</td>
<td>0.491*</td>
<td>0.191*</td>
</tr>
<tr>
<td></td>
<td>(23.48)</td>
<td></td>
</tr>
<tr>
<td>Imm</td>
<td>-0.168*</td>
<td>-0.066*</td>
</tr>
<tr>
<td></td>
<td>(-6.91)</td>
<td></td>
</tr>
<tr>
<td>Perm Inc</td>
<td>0.000002*</td>
<td>7.29e-06*</td>
</tr>
<tr>
<td></td>
<td>(31.51)</td>
<td></td>
</tr>
<tr>
<td>Trans Inc</td>
<td>0.000011*</td>
<td>4.09e-06*</td>
</tr>
<tr>
<td></td>
<td>(41.93)</td>
<td></td>
</tr>
<tr>
<td>Div Inc</td>
<td>0.574*</td>
<td>0.216*</td>
</tr>
<tr>
<td></td>
<td>(35.23)</td>
<td></td>
</tr>
<tr>
<td>Hhsize</td>
<td>0.030*</td>
<td>0.012*</td>
</tr>
<tr>
<td></td>
<td>(5.64)</td>
<td></td>
</tr>
<tr>
<td>Own Price</td>
<td>-8.54e-06*</td>
<td>-3.33e-06*</td>
</tr>
<tr>
<td></td>
<td>(-41.06)</td>
<td></td>
</tr>
<tr>
<td>Rent Price</td>
<td>0.002*</td>
<td>0.0009*</td>
</tr>
<tr>
<td></td>
<td>(22.02)</td>
<td></td>
</tr>
<tr>
<td>Nmov</td>
<td>0.692*</td>
<td>0.264*</td>
</tr>
<tr>
<td></td>
<td>(45.08)</td>
<td></td>
</tr>
<tr>
<td>Dis</td>
<td>-0.165*</td>
<td>-0.065*</td>
</tr>
<tr>
<td></td>
<td>(-6.33)</td>
<td></td>
</tr>
<tr>
<td>Edu</td>
<td>0.084*</td>
<td>0.033*</td>
</tr>
<tr>
<td></td>
<td>(4.52)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-2.670*</td>
<td>NA</td>
</tr>
<tr>
<td></td>
<td>(-45.10)</td>
<td></td>
</tr>
</tbody>
</table>

N: 48,464 | 32,511

NA: Not Applicable; z-statistics in parenthesis.

* indicates statistical significance at .01 or better. ** indicates statistical significance at .05.

3.2. Results of the Housing Demand Regressions for Homeowners. The results for the housing demand regressions for homeowners appear in Table 4.\textsuperscript{16} The statistically

\textsuperscript{16}As is typically done, owner-occupied demand is identified by assuming that within a geographic area and a given time period, suppliers cannot alter the level of housing services provided, i.e., the short-run housing supply curve is perfectly inelastic. This is consistent with the view that housing supply cannot immediately
significant demographic variables in both markets are Perm Inc, Trans Inc, Div Inc, Hhsize, Nmov, Dis and Edu. The Income, Hhsize and Edu variables are positive. Thus, households with higher incomes, who are less constrained in liquidity, larger households and households whose head has a higher educational attainment demand more owner-occupied housing. The Nmov and Dis variables are negative, so that demand for housing declines for households who do not move often and whose head is limited in amount of work. Sex and Age are statistically significant in the Atlanta housing market. Households in which the head of the household is female and older demand more owner-occupied housing. White and Mar are statistically significant in the San Francisco housing market and their coefficients suggest that households in which the head of the household is white and married demand more owner-occupied housing. In contrast with its important role in explaining tenure choice, the demographic variable Imm is not important in explaining actual demand for housing services.

The lambda variable is negative and statistically significant in both housing markets. This implies that unobservables which affect the probability of owning are significantly negatively correlated with those that affect the demand for housing services. Thus, ignoring the possible selection bias due to tenure choice and running OLS on the sample of homeowners would lead to inconsistent estimators.

adjust to a change in prices [Smith et al. (1988)]. Thus, controlling for demand shifters, price differences that are observed across geographic areas map out the demand curve.
Table 4. Housing Demand Regressions for Homeowners

<table>
<thead>
<tr>
<th>Variable</th>
<th>San Francisco</th>
<th>Coefficient</th>
<th>z-statistic</th>
<th>Coefficient</th>
<th>z-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sex</td>
<td>0.008</td>
<td>1.08</td>
<td></td>
<td>0.046*</td>
<td>2.91</td>
</tr>
<tr>
<td>Age</td>
<td>-0.0001</td>
<td>-0.14</td>
<td></td>
<td>0.007*</td>
<td>10.39</td>
</tr>
<tr>
<td>White</td>
<td>0.088*</td>
<td>12.82</td>
<td></td>
<td>-0.011</td>
<td>-0.59</td>
</tr>
<tr>
<td>Mar</td>
<td>0.025*</td>
<td>2.59</td>
<td></td>
<td>-0.001</td>
<td>-0.06</td>
</tr>
<tr>
<td>Imm</td>
<td>0.019</td>
<td>1.70</td>
<td></td>
<td>0.060</td>
<td>1.53</td>
</tr>
<tr>
<td>Perm Inc</td>
<td>3.89e-06*</td>
<td>18.48</td>
<td></td>
<td>0.00001*</td>
<td>23.61</td>
</tr>
<tr>
<td>Trans Inc</td>
<td>2.33e-06*</td>
<td>30.25</td>
<td></td>
<td>0.00001*</td>
<td>59.10</td>
</tr>
<tr>
<td>Div Inc</td>
<td>0.055*</td>
<td>7.93</td>
<td></td>
<td>0.125*</td>
<td>9.47</td>
</tr>
<tr>
<td>Hhsize</td>
<td>0.008*</td>
<td>3.92</td>
<td></td>
<td>0.012*</td>
<td>2.70</td>
</tr>
<tr>
<td>Own Price</td>
<td>-8.31e-07*</td>
<td>-15.29</td>
<td></td>
<td>8.00e-06*</td>
<td>11.44</td>
</tr>
<tr>
<td>Nmov</td>
<td>-0.094*</td>
<td>-12.24</td>
<td></td>
<td>-0.287*</td>
<td>-18.03</td>
</tr>
<tr>
<td>Dis</td>
<td>-0.035*</td>
<td>-3.18</td>
<td></td>
<td>-0.052**</td>
<td>-2.35</td>
</tr>
<tr>
<td>Edu</td>
<td>0.133*</td>
<td>17.83</td>
<td></td>
<td>0.189*</td>
<td>12.93</td>
</tr>
<tr>
<td>Constant</td>
<td>1.086*</td>
<td>33.52</td>
<td></td>
<td>-0.068</td>
<td>-1.05</td>
</tr>
<tr>
<td>Lambda</td>
<td>-0.161*</td>
<td>-9.74</td>
<td></td>
<td>-0.185*</td>
<td>-5.49</td>
</tr>
</tbody>
</table>

* indicates statistical significance at .01 or better. ** indicates statistical significance at .05. Lambda is the coefficient of the inverse Mill’s ratio.

3.3. The Housing Price Variable. We now examine the results for the housing price variable. We find that in the San Francisco housing market an increase in housing price leads to a statistically significant decrease in owner-occupied housing demand. In the Atlanta housing market, however, we find that housing price has a positive and statistically significant effect on the demand for housing services for those who choose to own.\(^\text{17}\) This result refutes traditional demand theory, but is fully consistent with the predictions of our model. It points to the relative strength of the expected capital gains effect that induces individuals to buy more owner-occupied housing services when its price increases\(^\text{18}\). Since housing is

\(^{17}\)Wealthy may buy high priced housing in high priced areas. Since we do not have a wealth variable in our data set, following many other studies in the housing literature, we use permanent income together with the liquidity constraint variable to represent potential lifetime earnings.

\(^{18}\)An alternative story goes as follows: an increase in the price of housing leads to an expectation of capital gain. The consumer discounts this amount and subtracts it from the current price, thereby having a "perceived" price. Hence, his demand may be increasing with respect to an increase in the current market price of housing (which is observable), but it is increasing as his perceived price declines.
both a consumption good and an investment asset, changes in its price can evoke both a traditional demand effect and a capital gains effect. The model set forth in Section 1, with its emphasis on intertemporal price expectations, captures the mechanism by which this occurs. In particular, it allows for the possibility that a sufficiently strong capital gains effect can cause the own-demand for owner-occupied housing to be upward sloping, even though housing is a non-inferior good. In this case, housing is quasi-Giffen.

The results with respect to the housing price variable reflect the economic conditions surrounding the California and Atlanta economies in the late 1980s and early 1990s. California witnessed a significant recession during the early 1990s. Moreover, California’s homeownership rate (55.6%) was one of the lowest in the nation in 1990 [California Department of Housing and Community Development (1998)]. Homeownership rates along the coastal areas such as the San Francisco housing market also tended to be lower. The San Francisco housing market in the late 1980s and early 1990s was therefore not strong and thus one would not expect the relative dominance of the capital gains effect in this market. Atlanta, on the other hand, witnessed an economic expansion in the late 1980s and early 1990s reinforced by employment growth and housing permit growth. The Atlanta housing market was very strong in this era (and is still strong) because of its economy’s ability to attract high-skilled workers, competitive business costs and the existence of high technology industries. Thus, one would expect the relative dominance of the capital gains effect in the Atlanta housing market that might induce individuals to buy more owner-occupied housing services when its price increases.
4. Summary and Conclusions

Since owner-occupied housing is as much an investment asset as it is a consumption good, its own-demand curve properties may differ from the traditional predictions associated with standard consumption goods. In particular, housing offers the possibility of a change in capital value. An increase in its price, when taken as a signal for expectations of capital gain, could lead to an increase in the demand for housing services. When this capital gains effect outweighs the traditional neoclassical demand response, the own-demand curve for housing would be upward sloping, even though housing is non-inferior. This paper studies this possibility (that housing can sometimes be a quasi-Giffen good), both theoretically and empirically.

In the theoretical part of the paper, we use a two-period life cycle model with uncertainty to derive testable predictions that the own-demand functions for the standard consumer goods have the traditional downward sloping property while allowing for the possibility that the own-demand function for housing may be upward sloping. This makes for a clear theoretical distinction between housing and the other consumer goods. In the empirical part of the paper, we estimate tenure choice and housing demand equations for PUMA samples from two large metropolitan areas, San Francisco and Atlanta, and find that housing demand is downward sloping in the former but upward sloping in the latter, indicating the presence of a capital gains effect in Atlanta that is sufficiently strong as to dominate the standard neoclassical response. Our theoretical model reconciles these apparently conflicting results and provides a rationale for each of the findings.
Appendix

The Slutsky system in (5) can be rewritten as

\[
\frac{\partial q_i}{\partial p_k} + q_k \frac{\partial q_i}{\partial Y} = \lambda \frac{D_{ki}}{D} - \sum_{j=1}^{n+1} V_{qjp_k} \frac{D_{ji}}{D} = S_{ik}, \quad \forall \ i, k,
\]

where we have grouped economic observables on the left hand side of the equality and unobservables on the right. The unobservables of the Slutsky system in (A1) can be written in matrix form as

\[
S = \frac{\lambda}{D} \left[ \begin{array}{cccccc}
D_{11} & D_{12} & \ldots & D_{1n} & D_{1,n+1} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
D_{n+1,1} & D_{n+1,2} & \ldots & D_{n+1,n} & D_{n+1,n+1}
\end{array} \right] + \frac{1}{D} \left[ \begin{array}{cccc}
V_{x_1p_1} & V_{x_2p_1} & \ldots & V_{x_np_1} \\
\vdots & \vdots & \ddots & \vdots \\
V_{x_1p_n} & V_{x_2p_n} & \ldots & V_{x_np_n}
\end{array} \right] \times
\left[ \begin{array}{cccc}
D_{11} & D_{12} & \ldots & D_{1n} \\
\vdots & \vdots & \ddots & \vdots \\
D_{n+1,1} & D_{n+1,2} & \ldots & D_{n+1,n+1}
\end{array} \right] \frac{1}{D} \left[ \begin{array}{cccc}
V_{h_1p_1} & V_{h_2p_1} & \ldots & V_{h_np_1} \\
\vdots & \vdots & \ddots & \vdots \\
V_{h_1p_n} & V_{h_2p_n} & \ldots & V_{h_np_n}
\end{array} \right].
\]

This can be expressed as

\[
S = \frac{(\lambda \Delta + F \Delta)}{D}.
\]

We partition this matrix so that we get

\[
S = \left[ \begin{array}{cc}
S_{1}^{1,n \times n} & S_{1}^{2,n \times 1} \\
S_{1}^{3,n \times n} & S_{1}^{4,n \times 1}
\end{array} \right] = \frac{\lambda}{D} \left[ \begin{array}{cc}
\Delta_{1}^{1,n \times n} & \Delta_{1}^{2,n \times 1} \\
\Delta_{1}^{3,n \times n} & \Delta_{1}^{4,n \times 1}
\end{array} \right] + \frac{1}{D} \left[ \begin{array}{cc}
F_{1}^{1,n \times n} & F_{1}^{2,n \times 1} \\
F_{1}^{3,n \times n} & F_{1}^{4,n \times 1}
\end{array} \right] \left[ \begin{array}{cc}
\Delta_{1}^{1,n \times n} & \Delta_{1}^{2,n \times 1} \\
\Delta_{1}^{3,n \times n} & \Delta_{1}^{4,n \times 1}
\end{array} \right],
\]

which equivalently can be written as

\[
S = \left[ \begin{array}{cc}
S_{n \times n}^{1} & S_{n \times 1}^{2} \\
S_{n \times n}^{3} & S_{n \times 1}^{4}
\end{array} \right] = \frac{\lambda}{D} \left[ \begin{array}{cc}
\Delta_{n \times n}^{1} & \Delta_{n \times 1}^{2} \\
\Delta_{n \times n}^{3} & \Delta_{n \times 1}^{4}
\end{array} \right] + \frac{1}{D} \left[ \begin{array}{cc}
F_{n \times n}^{1} & F_{n \times 1}^{2} \\
F_{n \times n}^{3} & F_{n \times 1}^{4}
\end{array} \right] \left[ \begin{array}{cc}
\Delta_{n \times n}^{1} & \Delta_{n \times 1}^{2} \\
\Delta_{n \times n}^{3} & \Delta_{n \times 1}^{4}
\end{array} \right],
\]

where \( S_{n \times n}^{1} \) is the submatrix of the Slutsky unobservables which relate each consumer good with the consumer good prices, \( S_{n \times 1}^{2} \) is the submatrix of the Slutsky unobservables which relate owner-occupied housing services with the consumer good prices, and \( S_{1 \times n}^{3} \) and \( S_{1 \times 1}^{4} \)
consists of the Slutsky unobservables which relate the price of owner-occupied housing with the consumer goods and owner-occupied housing services. We put forth the following proposition:

**Proposition 1.** Let \( S \) be the \((n+1) \times (n+1)\) matrix of Slutsky unobservables. Suppose we partition this matrix so that \( S_{n \times n}^1, S_{n \times 1}^2, S_{1 \times n}^3 \) and \( S_{1 \times 1}^4 \) are the submatrices. Assume that

\[
\frac{\lambda}{D} \Delta_{nn}^1 + \frac{1}{D} \left[ F_{n \times n}^1 \Delta_{n \times n}^1 + F_{n \times 1}^2 \Delta_{1 \times n}^3 \right],
\]

\( F_{n \times n}^1 \Delta_{n \times n}^1 \) is a symmetric submatrix and that \( V_{xpc} = f(x,p_c) \).\(^{19}\) Then a general form of \( V(x,h^o,p_c, p_o, p_r) \) is

\[
(A2) \quad V(x,h^o,p_c, p_o, p_r) = \Pi(x,h^o,p_o,p_r) + \sigma(p_c, p_o, p_r) + K \sum_{i=1}^{N} p_i x_i,
\]

where \( p_c = (p_1, ..., p_n) \) is the vector of consumer good prices; \( \Pi(.) \) is any real valued twice continuously differentiable function, increasing in \( x \) and \( h^o \); \( \sigma(.) \) is any real valued twice continuously differentiable function, and \( K \) is any non-negative constant.

**Proof of Proposition 1:**

**Step 1:** The imposition of symmetry on the submatrix \( F_{n \times n}^1 \Delta_{n \times n}^1 \) holds if

\[
(A3) \quad \frac{\partial^2 V}{\partial x_j \partial p_i} = \delta_{ij} \phi(x,p_c), \quad i, j = 1, ..., n,
\]

where

\[
\delta_{ij} = \begin{cases} 
1 & \text{if } i = j \\
0 & \text{otherwise.}
\end{cases}
\]

\(^{19}\) The effect of a consumer good price change on the expected marginal utility of any consumption good depends on all consumer goods and their prices, it is independent of owner-occupied housing service, the price of owner-occupied housing and the price of rental housing.
We need to construct a solution to the system of partial differential equations in (A3). From Dusansky and Wilson (1993), we know that such a solution is

\[ V = K \sum_j p_j x_j + \bar{\sigma}(p_c) + \bar{\Pi}(x). \]

**Step 2:** Since there are no restrictions on \( S_{n \times 1}^2, S_{1 \times n}^3 \) and \( S_{1 \times 1}^4 \), we obtain the following two kinds of systems of partial differential equations:

(A4) \[ \frac{\partial^2 V}{\partial q_j \partial p_o} = h_{jo}(x, h^o, p_c, p_o, p_r), \]

for \( j = 1, 2, ..., n + 1 \), and

(A5) \[ \frac{\partial^2 V}{\partial q_j \partial p_i} = h_{ji}(x, h^o, p_c, p_o, p_r), \]

for \( j = n + 1 \) and \( i = 1, 2, ..., n \). A solution to (A4) is of the form

(A6) \[ V^o = \varphi^o(x, h^o, p_c, p_o, p_r) + k^o(p_c, p_o, p_r) + \delta^o(x, h^o, p_1, ..., p_n, p_r). \]

Similarly, a solution to (A5) is of the form

(A7) \[ V' = \varphi'(x, h^o, p_c, p_o, p_r) + k'(p_c, p_o, p_r, x_1, ..., x_n) + \delta'(x, h^o, p_o, p_r). \]

Combining (A6) and (A7) puts the following restrictions on \( V \): \( \delta^o(x, h^o, p_1, ..., p_n, p_r) \) must be independent of \( p_1, ..., p_n \), \( k'(p_c, p_o, p_r, x_1, ..., x_n) \) must be independent of \( x_1, ..., x_n \), and \( \delta'(x, h^o, p_o, p_r) \) must be independent of \( p_o \). Thus, employing these restrictions we obtain

(A8) \[ \hat{V} = \varphi(x, h^o, p_c, p_o, p_r) + \hat{k}(p_c, p_o, p_r) + \hat{\delta}(x, h^o, p_r). \]

**Step 3:** We now combine \( \bar{V} \) obtained in step one and \( \hat{V} \). \( \bar{V} \) puts some restrictions on \( \varphi(x, h^o, p_c, p_o, p_r) \). Since

\[ \frac{\partial^2 \bar{V}}{\partial x_j \partial p_i} = \begin{cases} K & \text{if } i = j \\ 0 & \text{otherwise} \end{cases} \]

for all \( i, j = 1, ..., n \), we must have that \( p_c = (p_1, ..., p_n) \) and \( x \) is separable except for the term \( \sum_j p_j x_j \). Thus, we get \( \varphi(x, h^o, p_c, p_o, p_r) = \varphi(x, h^o, p_o, p_r) \). Combining \( \bar{V} \) and \( \hat{V} \) under
this restriction yields

\[(A9) \quad V = \Pi(x, h^o, p_o, p_r) + \sigma(p_c, p_o, p_r) + K \sum_j p_j x_j,\]

which is equation (A2). \(\square\)

In view of (A2), the Slutsky expression for the own-demand curve for the consumption goods is given by

\[(A10) \quad \frac{\partial x_i}{\partial p_i} = -x_i \frac{\partial x_i}{\partial Y} + (\lambda - K) \frac{D_{i,i}}{D}, \forall i = 1, ..., n.\]

By the second order conditions, we know that \(\frac{D_{i,i}}{D} < 0\). Hence, we have the result that if a consumption good is a non-inferior commodity (i.e., if \(\frac{\partial x_i}{\partial Y}\) is positive), then \(\frac{\partial x_i}{\partial p_i} < 0.\)\(^{20}\) This is the traditional property of a downward-sloping own-demand curve for the consumption goods. In view of (A2), the Slutsky expression for the own-demand curve for owner-occupied housing derived in (6) does not change. Thus, we recover the traditional Slutsky-Hicks properties for the consumer goods while simultaneously allowing for an upward-sloping own-demand curve for housing.

\(^{20}\)We know that \(\lambda > 0\), by the construction of the Lagrangian, and that \(K\) can be arbitrarily small. Thus, \((\lambda - K) > 0\).
References


