Are Empirical Measures of Macroeconomic Uncertainty Alike?

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Abstract
This paper considers a plethora of time-series measures of uncertainty for inflation and real output growth, which are widely used in empirical studies. Their relative performances are compared to a benchmark measure using the uncertainty measure reported by individual forecasters in the Survey of Professional Forecasters (SPF) for the period 1982-2008. The results show that the use of real-time data with fixed-sample recursive estimation of an asymmetric bivariate GARCH model produces inflation uncertainty estimates which replicate the survey measure more closely than any other time-series models. There is, however, overwhelming evidence that many of the time series measures of growth uncertainty overestimate the level of benchmark survey measure. The implications of our results are discussed in the context of the extensive empirical studies on macroeconomic uncertainty.

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1. Introduction

A large body of macroeconomic research requires identifiable measures of, or assumptions about, economic uncertainty in its attempts to uncover certain underlying relationships between a set of economic variables. Following Keynes’s *General Theory* (1937), the notion and hence role of economic uncertainty has been a key underpinning factor in both theory and policy discussions. In the *General Theory*, Keynes recognises that some things are more predictable than others. Consumption, postulated as a function of current income, is perceived to be more stable than investment, which depends on expected future demand or “animal spirits”, as well as the interest rate.¹ He also believes that a great part of economic instability (or uncertainty) is due to a failure to communicate information about the future. In the *General Theory* he explains how additional savings could depress current demand because they fail to signal a future demand for consumption to compensate for loss of demand in consumption today. Consequently, investors who base their investment decisions on expected future demand, which is proxied by current demand, may obtain the wrong message about future prospects and potentially reduce production capacity. The concept of economic uncertainty, which in central to Keynes’s work, is a result of a lack of information or deficient foresight.

This theme of economic uncertainty has continued to reverberate in a plethora of economic research focusing, in particular, on macroeconomic variables like inflation and output growth. Since Friedman’s (1977) proposition that increased inflation uncertainty may adversely affect real economic variables, a growing body of research which explores the real effects of economic uncertainty has evolved in the last couple of decades. In this stream of work, economists have addressed questions as diverse as the effects of macroeconomic uncertainty on a country’s macroeconomic performance (Grier and Perry, 2000), contract duration (Rich and Tracy, 2004), firms’ investment rate (Beaudry et al., 2001), the current account (Ghosh and Ostry, 1997) and banks’ loanable funds allocation (Baum et al., 2009). To empirically test the effects of macroeconomic uncertainty on a variable of interest, it is common to use uncertainty measures obtained from surveys and/or time series models because of the

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¹ Minsky (1975) provides further reasons why investment could be an unstable function of current income.
lack of direct observations on inflation and output growth uncertainty. It comes as no surprise, therefore, that a myriad of uncertainty measures have emerged in the literature. Notwithstanding this, little is known about the quality of these uncertainty measures and their relative merits, nor is there any consensus about how best to estimate measures of macroeconomic uncertainty. Given that any empirical assessment of the effects of economic uncertainty hinges on the quality of uncertainty measures used, an evaluation of various proxies for macroeconomic uncertainty assumes great importance. This calls for an evaluation of the current empirical models used as models to produce measures of macroeconomic uncertainty.

In this paper, we focus on a wide spectrum of empirical models which produce inflation and output growth uncertainty proxies. The relative performances of these proxies are then compared to a benchmark measure using the uncertainty measure reported by individual forecasters in the Survey of Professional Forecasters (SPF). In doing so, this paper makes four important contributions on the construction of uncertainty measures which are highly relevant for empirical research. First, we show that special attention should be paid to choosing an appropriate data set when constructing inflation and real growth uncertainty proxies. There is a predominance of using revised data to generate measures of macroeconomic uncertainty in the empirical literature. While revised data comprising of realised inflation and real growth rates reflect the state of the economy more accurately because of data revisions over time, this data set is not available to a forecaster in real time. In contrast, the real-time data set reflects, at each date, exactly what the macroeconomic data look like at that date. Such a feature of real-time data implies that the data contain information structure that is more realistic to forecasters in predicting future levels of inflation and real growth, and hence their uncertainties. Moreover, uncertainty measures generated by the use of real-time data can be regarded as *ex ante* measures that are comparable to the survey measure. An interesting result emerges, that is regardless of the models used to generate macroeconomic uncertainty proxies, the measure obtained from real-time data tends to be more highly correlated with the survey measure compared with proxies obtained from revised data.\(^2\) These results

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\(^2\) Macroeconomic uncertainty which is constructed from revised data is considered an *ex post* measure because the conditioning information set comprises of realised inflation and real growth data that are not available to a forecaster in real-time.
clearly emphasize the need for empirical researchers to employ real-time data, wherever available, rather than revised data in forecasting uncertainty as the proxies generated by these two data sets can differ significantly.

Second, we consider pertinent features of time series models which will produce uncertainty proxies that better track the benchmark survey measure. The literature is divided when it comes to choosing an appropriate model for generating macroeconomic uncertainties. Although there is a proliferation in the use of the generalised autoregressive conditional heteroskedasticity (GARCH) model to proxy for inflation and output growth uncertainty, such a model has been criticised for its lack of economic intuition and theoretical foundation (Peng and Yang, 2008). The bivariate GARCH model of inflation and output growth, unlike the structural vector autoregressive (SVAR) models, fails to impose any restrictions to properly identify the underlying structural shocks thus clouding the economic interpretation of these shocks and their associated uncertainty. Proponents of the GARCH model, however, argue that the model is attractive from a statistical standpoint due to the time-varying property of the uncertainty proxy represented by the conditional variance (or standard deviation) of the unpredictable innovations in inflation and real growth (Grier and Perry, 2000). In addition, the effects of differently signed shocks on uncertainty can be easily parameterised using an asymmetric GARCH model. We show that there is a trade-off between models that properly identify the structural shocks and models that account for the heteroskedasticity in the innovations of inflation and real growth. Our results are generally in favour of models which explicitly specify the observed heteroskedasticity in inflation and growth innovations as they generate uncertainty proxies that are more consistent with the survey measure.

Third, within the class of GARCH, vector autoregressive (VAR) and SVAR models we examine whether an informatively richer model which allows for interactions between inflation and output growth, and controls for the effects of monetary policy produces uncertainty proxies that better match the survey measure. The results point to evidence in support of an informatively richer model. Within the class of GARCH models, we find that the bivariate GARCH model produces superior inflation uncertainty proxy to the univariate GARCH models. As for the VAR and SVAR models, there is evidence that a model which incorporates the long- and short-
term interest rates generate uncertainty proxies that closely follow movements of the survey measure. On the other hand, we find that irrespective of the models and data used to produce growth uncertainty proxy they tend to overestimate its level compared with the survey measure.

Last but not least, in respect of the GARCH models we identify whether a model that accounts for features such as asymmetric conditional variance in inflation and growth innovations, levels dependence in the conditional variance of inflation shocks, and structural breaks in the mean process would improve their performance in generating uncertainty proxies that better track the survey measure. Of the three features examined, we find that the asymmetric conditional variance is the most important feature that must be accounted for in the model specification to improve the forecasting performance of inflation uncertainty. The proposed fixed-sample recursive estimation method is also proven to deliver inflation uncertainty proxy that match the survey measure more closely than the time-varying GARCH model.

The rest of the paper is structured as follows. Section 2 explains the various methodologies used to proxy macroeconomic uncertainty. Section 3 describes the data. Section 4 evaluates the different measures of uncertainty by comparing them with the survey measure. The last section summarises our findings.

2. Empirical and Survey Measures of Economic Uncertainty

We employ two recursive estimation procedures on real-time and revised data to construct empirical measures of uncertainty. This recursive procedure reproduces the information structure that is available to forecasters (Giordani and Soderlind, 2003). The first procedure starts with a sample of 15 years data (1966Q4-1981Q4) and estimates the model recursively by adding one observation at a time until the whole sample of data (up till 2008Q3) is fully utilised. The second procedure fixes the sample period to 15 years of data and estimates the model recursively over the sample period with the latest 15 years of data. The sample in the second procedure

3 Although there is no guide to what exactly constitutes a sufficiently long period of data, we arbitrarily choose 15 years since this is consistent with the sample period used by Giodarni and Soderlind (2003).
does not become progressively larger. One reason for adopting these two distinct procedures is that while the two procedures potentially account for possible structural change in the underlying mean process which in turn affects the forecasts of uncertainty, the second procedure does so more accurately. By taking into consideration the latest 15 years of data, any potential regime shift is fully accounted for at some stage in the recursive estimation process and their effects are not persistently captured by including past observations. Accounting for possible structural changes is vital as it is widely acknowledged that the U.S. output growth volatility (and therefore uncertainty) has decreased significantly in the mid 1980s, a phenomenon commonly known as the Great Moderation (Fang and Miller, 2008 and references therein).

2.1 The Survey Measure

Survey data are commonly employed to provide uncertainty proxies as they provide direct measures of inflation and output growth expectations, which circumvents possible errors in specifying how people form their forecasts. We employ the model of Giodarni and Soderlind (2003) to proxy for macroeconomic uncertainty. An attractive feature of their model is that they account for the dispersion of different forecasters’ probability distributions – an improvement over the degree of disagreement among point forecasts by different forecasters. As first pointed out by Zarnowitz and Lambros (1987), high dispersion of point forecasts should not be interpreted as indicating high uncertainty.

Giodarni and Soderlind (2003) specify the information set of forecaster $i$ by a scalar signal $z_i$ and denote the probability density function of inflation conditional on receiving the signal of forecaster $i$ as $pdf(\pi \mid i)$. Assuming that $\pi$ and $z_i$ are random variables, and the latter having density function $pdf(i)$, the aggregate distribution function can be written as

$$pdf_{\lambda}(\pi) = \int_{-\infty}^{\infty} pdf(\pi \mid i) pdf(i) di. \quad (1)$$
Equation (1) resembles, but it is not, the marginal distribution of $\pi$. This is because by combining the density forecasts of each forecaster, the information sets are pooled together instead of being integrated out to yield a marginal density. In practice, it is likely that forecasters have diverse information sets but with much commonly shared public information. Hence, the representation of this complex information using a scalar random variable and the aggregation over forecasters with a well-defined distribution as in (1) may be deemed as unrealistic (Wallis, 2005). Hence, this statistical framework for the survey measure which utilises the dispersion of different forecasters’ probability distributions, provides a more robust approach compared with the disagreement measure of point forecasts.

On the basis of equation (1) and using standard relation between the variances of conditional and marginal distributions, it can be shown that the variance of the aggregate distribution is

$$ Var_{A}(\pi) = E(\sigma_i^2) + Var(\mu_i), \quad (2) $$

for which the left hand term is the variance of the survey aggregate histogram, and the first (second) term on the right hand side is the mean of individual uncertainty (the variance of the point forecast which measures the disagreement among forecasters). An important conclusion drawn from their evaluation of the individual density forecasts is that forecasters underestimated uncertainty. They show, on the other hand, that the aggregate standard deviation, $Std_{A}(\pi)$, and the mean of individual uncertainty, $E(\sigma_i)$, provide accurate confidence intervals which suggest that they can be used as reliable measures of uncertainty.\(^4\) In this paper, we adopt $E(\sigma_i)$ as our benchmark survey measure of uncertainty.

To compute $E(\sigma_i)$ we fit normal distributions to each histogram. The mean and variance are then estimated by minimizing the sum of squared difference between the survey probabilities and the probabilities for the same intervals implied by the

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\(^4\) The idea of correct unconditional confidence bands is to use the survey data to construct confidence interval around the point forecasts and determine if the 95% confidence intervals cover 95% of the actual outcomes of GDP deflator inflation. However, this unconditional measure assumes that the innovation process is identically and independently distributed.
normal distribution. The use of a normal distribution to approximate each forecast histogram is motivated by observations that, by and large, the forecast histograms look fairly symmetrical and that relatively greater probability mass in each interval is located close to its overall mean. Figures 1 and 2 show the aggregate histogram for inflation and output growth for all first quarters of 1982-2008, respectively. Note that the use of a normal distribution to approximate each forecast density differs from the assumption of a uniform distribution employed by Zarnowitz and Lambros (1987), Lahiri and Teigland (1987) and Diebold et al. (1999). As it can be seen in both figures, particularly for inflation (in Figure 1), the aggregate probabilities for different inflation and output growth rates largely appear to be symmetrical, thus fitting a uniform distribution is prone to overestimate the variances of inflation and output growth.

2.2 Univariate Time Series Approach

2.2.1 Univariate ARMA-GARCH Models

Univariate time series models of inflation uncertainty (Engle, 1983; Holland, 1995; Cosimano and Jansen, 1988) and output growth uncertainty (Caporale and McKiernan 1998; Fountas and Karanasos, 2007; Fang and Miller, 2008) are widely employed. There are several interesting features of inflation and output growth uncertainty. Henry et al. (2007) show that inflation rates and inflation uncertainty are tightly linked and that inflation uncertainty tends to exhibit an asymmetric response to unanticipated positive inflation shock than a negative shock of equal magnitude. Similarly, Henry and Olekalns (2002) document this asymmetric response in growth uncertainty; they find that an unanticipated negative growth shock elicits greater growth uncertainty than an equally sized positive shock. To accommodate these empirical features, the mean and variance specifications of inflation and output growth are modelled as:
\[ x_t = z_{t-1}\beta + \varepsilon_t, \quad \varepsilon_t \sim N(0, h_t) \] (4)

\[ h_t = \omega + \alpha(L)h_{t-1} + \beta(L)\varepsilon_{t-1}^2 + \gamma(L)\xi_{t-1}^2 + \delta\tilde{x}_{t-1} \] (5)

where \( x_t \) denotes output growth (\( \Delta y_t \)) and inflation (\( \pi_t \)). Here, \( z_{t-1} = [1, x_{t-1}, \ldots, x_{t-k}] \) and \( \beta \) is a vector of parameters corresponding to \( z_{t-1} \). An AR(4)-GARCH(1,1) model is fitted to the data following Giodarni and Soderlind (2003). In equation (5), we define \( \xi_{t-1} = \min(0, \varepsilon_{t-1}) \) for output growth and \( \xi_{t-1} = \max(0, \varepsilon_{t-1}) \) for inflation. This term takes into account the asymmetric response of uncertainty to equally sized but differently signed shocks. The term \( \tilde{x}_{t-1} \) is defined as \( \max(0, x_{t-1}) \) and it is only applicable to inflation uncertainty as it accounts for the widely observed levels dependence in inflation uncertainty. The conditional standard deviation \( \sqrt{h_t} \) is our measure of uncertainty. Notice that the uncertainty estimate at each point in time varies not only because of the asymmetric GARCH structure and levels dependence, but it also varies due to the recursive nature of the estimation procedure. Equations (4) and (5) are jointly estimated with the maximum likelihood method.

Equations (4) and (5) nest the symmetric and asymmetric GARCH models. We estimate a simple GARCH model of inflation and output growth with no asymmetry and levels dependence by restricting \( \gamma(L) = \delta = 0 \). In addition, asymmetric inflation and growth uncertainty measures are obtained from estimating equations (4) and (5) with the restriction \( \delta = 0 \). We compare and contrast these different uncertainty measures to identify the significance of capturing certain empirical regularities in forecasting growth and inflation uncertainty.

2.2.2 The Time-Varying GARCH Approach

While the recursive procedures account for possible structural breaks in the mean inflation and growth process, they do not impose a specific structure on the parameter vector \( \beta \). The parameter vector \( \beta \) would vary as and when an additional observation is added to the sample recursively. Evans (1991), on the other hand,
proposes a time-varying parameter model of inflation where variations in the mean inflation process can contribute to inflation uncertainty. He argues that changes in the private sector behaviour, economic policy and institutions can bring about changes in the mean inflation process. To determine whether the time varying GARCH approach produces a superior uncertainty proxy compared with the recursive approach, equation (4) is replaced with

\[ x_t = z_{t-1} \beta_t + \varepsilon_t, \quad \varepsilon_t \sim N(0, h_t) \]  

\[ \beta_t = \beta_{t-1} + v_t, \quad v_t \sim N(0, Q) \]

where \( v_t \) is a vector of normally distributed shocks to the parameter vectors \( \beta_t \) with a homoskedastic covariance matrix \( Q \). Equations (5), (6) and (7) describe a time-varying AR process with a GARCH specification for the shocks of inflation.

The effects of variations in the structure of inflation on uncertainty can be analysed using the Kalman Filter. The filtering equations are

\[ x_t = E_{t-1} \beta_t + \eta_t \]  

\[ H_{t-1} = z_{t-1} \Omega_{t-1} z_{t-1}^\prime + h_{t-1} \] 

\[ E_{t-1} \beta_t = E_{t-1} \beta_t + \left[ \Omega_{t-1} z_{t-1}^\prime H_{t-1}^{-1} \right] \eta_t \] 

\[ \Omega_{t+1|t} = \left[ I - \Omega_{t-1} z_{t-1}^\prime H_{t-1}^{-1} z_{t-1} \right] \Omega_{t|t-1} + Q. \]  

Uncertainty about the structure of the inflation process, which is given by the variance-covariance matrix of \( \beta_t \) conditioned on information available at period \( t-1 \), is denoted as \( \Omega_{t|t-1} \). As it can be seen in equation (8), given that the innovations in inflation \( \eta_t \) may arise from inflation shocks \( \varepsilon_t \) and unanticipated changes in the
structure of inflation \( v_t \), the conditional variance of inflation \( H_{t-1} \) therefore depends upon both \( h_{t-1} \) and the conditional variance of \( z_{t-1} \beta_t \) which is \( z_{t-1} \Omega_{t|t-1} z_{t-1}' \) (see equation 9). Note that the constant parameter model (4) is a special case of the time-varying parameter model. In the absence of uncertainty about \( \beta_t \), \( \Omega_{t|t-1} \) is a null matrix and the dynamics of \( h_{t-1} \) fully governs the conditional variance of inflation. It can also be inferred that wrongly fitting a constant parameter model when there is uncertainty about \( \beta_t \) will tend to understate the true conditional variance \( H_{t-1} \) by \( z_{t-1} \Omega_{t|t-1} z_{t-1}' > 0 \). We estimate a time varying AR(1)-GARCH(1,1) model non-recursively using revised data for inflation and output growth. Given that the time-varying GARCH model itself accounts for possible structural breaks in the mean process, it does not make sense to perform the recursive estimation procedure with this model. Empirical measures of inflation and growth uncertainty are obtained from their respective \( \sqrt{h_t} \) estimates.

2.3 Multivariate Time Series Approach

2.3.1 The SVAR approach

The SVAR approach is widely used in policy analysis as it uncovers and provides economic interpretations to the structural shocks. To determine whether an SVAR approach provides uncertainty estimates that are superior to the VAR and GARCH models, we estimate the following small scale macroeconometric model

\[
\begin{bmatrix}
\Delta y_t \\
\pi_t \\
i_t^f \\
i_t^l
\end{bmatrix}
= 
B
\begin{bmatrix}
\Delta y_{t-1} \\
\pi_{t-1} \\
i_{t-1}^f \\
i_{t-1}^l
\end{bmatrix}
+ 
\begin{bmatrix}
\varepsilon_{y_t}^y \\
\varepsilon_{\pi_t}^z \\
\varepsilon_{i_t^f}^{mp} \\
\varepsilon_{i_t^l}^l
\end{bmatrix}.
\]

(13)

We attempted to estimate a time-varying AR(3)-GARCH model but the estimation procedure fails to converge.
Equation (13) is analogous to a standard VAR model of monetary policy (MP) with one noticeable difference – it incorporates a long-term interest rate \( i_t^L \) which captures expectations of future inflation and future monetary policy. The inclusion of such a variable in the VAR model helps in the identification of the true underlying policy shocks more accurately (Rudebusch, 1996; Bagliano and Favero, 1999).\(^6\) We identify our empirical model by imposing the restriction that the matrix \( A \) be lower triangular with unit diagonal elements. Given that the long and short rates appear at the bottom of the system, the identification strategy presupposes that innovations in the long and short rates influence both real and nominal variables with a one-period lag. Our identification scheme can be justified by the fact that decisions that will affect production and output growth take time to plan and implement, and there is usually a lagged effect of monetary policy on inflation.

Another noteworthy point of our model is that given the identification of MP shock, the shock to \( i_t^L \) can be interpreted as a yield spread shock through the relation 

\[ sp_t = i_t^L - i_t^{ff} \]

where the federal funds rate is the short rate. This can be visualised by substituting the long rate in equation (13) using the yield spread definition 

\[ sp_t = i_t^L - i_t^{ff} \]. Since \( i_t^{ff} \) appears as a regressor in the last equation of the VAR system, the estimated residual of the newly defined last equation is equivalent to \( e_t^L \).

As documented by Bernanke and Blinder (1992), Estrella (2005), Estrella and Hardouvelis (1991), Plosser and Rouwenhorst (1994) among others, the yield spread contains significant information as a predictor of future growth and inflation rates. With an identified MP shock, the SVAR model can then separate out the MP shocks from market expectations. In sum, this model which does not impose a long-run restriction but incorporates inflationary expectations through the inclusion of a long-term interest rate provides a useful alternative to the Gali (1992) and Blanchard-Quah (1989) type SVAR models that were used in previous studies such as Rich and Tracey (2004).

\(^6\) Rudebusch (1996) highlights a common pitfall in the identification of monetary policy shocks in VAR models. Often innovations obtained from a time-invariant, linear reaction function of monetary authority which reacts only to a limited set of variables included in the model, may bear little or no relation with the true underlying shocks. As a remedy, he favours the direct use of expectations of future monetary policy actions embodied in some financial prices (Bagliano and Favero, 1999).
2.3.2 The VAR Approach

We estimate a four-variable VAR model comprising GDP deflator inflation, real GDP growth, the federal funds rate, and a 3-year interest rate. The standard deviation of the VAR forecast errors of GDP deflator inflation and real GDP growth are employed as measures of real and nominal uncertainty, respectively. In addition, we estimate a bivariate VAR model of Cecchetti and Rich (2001) and Peng and Yang (2008) that involves real GDP growth and GDP deflator inflation. For both models, a VAR(4) is estimated. There are reasons for estimating this simpler VAR specification. First, this simple bivariate VAR specification will be compared with the recursive bootstrapped VAR approach of Peng and Yang (2008). Unlike the recursive VAR approach that assumes homoskedastic errors in inflation and output growth, Peng and Yang (2008) controls for the presence of heteroskedastic errors in inflation and output growth.7 Secondly, the four-variable VAR model which accounts for the effects of monetary policy, is a richer model than the bivariate VAR specification. By comparing uncertainty measures obtained from these two models, we can assess whether an informatively richer model is required to generate a more accurate measure of macroeconomic uncertainty. Thirdly, we can compare uncertainty measures obtained from the four-variable VAR model and the SVAR model to determine whether the identification of structural shocks helps improve the performance of uncertainty forecasts.

2.3.3 The Recursive Bootstrapped VAR Approach

Peng and Yang (2008) propose an alternative method of generating a time-varying measure of uncertainty. They propose running a VAR model

\[ Z_t = B(L)Z_{t-k} + u_t \]  

(14)

7 The fact that inflation uncertainty peaks with the level of inflation suggests that inflation error is not likely to be homoskedastic.
where \( Z_i = [\Delta y_i, \pi_i] \), \( u_i = [u_{y_i}, u_{\pi_i}] \) and \( B(L) \) is a \( 2 \times 2 \) matrix of polynomial lags. Estimating a VAR model with an optimal lag length purges any possible serial correlation in the innovations. However, the presence of volatility clustering in the innovations of both output growth and inflation, which is commonly observed in the data, would give rise to an inconsistent parameter standard error estimates. Wu (1986) and Liu (1988) propose a bootstrap re-sampling procedure that provides a consistent estimator for the standard error of the parameter estimates when the error variances are heteroskedastic. This method is particularly useful when measuring uncertainty using the VAR forecast errors.

Wu (1986) and Liu (1988) suggest a weighting scheme on the VAR residuals to circumvent their non-identical and independent distribution. Having estimated (14), in our case a VAR(4), the residuals are obtained as

\[
 u_i = Z_i - \hat{Z}_i. \tag{15}
\]

These residuals are multiplied with an adjustment factor \( \lambda_i \) to form an empirical distribution function, \( \hat{F}_i \). Peng and Yang (2008) employ a discrete distribution that puts mass on two points \( a_i = a \cdot \hat{u}_i \) and \( b_i = b \cdot \hat{u}_i \) such that the empirical distribution follows \( \hat{F}_i = p \delta_{a_i} + (1 - p) \delta_{b_i} \). \( \delta_x \) denotes a probability measure which places a unit mass at \( x \) and \( p \in [0, 1] \). For the proposed method to be operational, the parameter values \( a, b \) and \( p \) need to be computed. These values are obtained by solving the system of equations obtained from the conditions laid out by Liu (1988) that is,

\[
 E(\lambda_i) = 0, \quad Var(\lambda_i) = 1 \quad \text{and} \quad E(\lambda_i^2) = 1. \]

The first two conditions are sufficient for proving the consistency of the bootstrap, while the third condition is sufficient for correcting the skewness term in the sampling distribution of the parameter estimates. It can be shown that \( a = (1 - \sqrt{5})/2 \), \( b = (1 + \sqrt{5})/2 \) and \( p = (5 + \sqrt{5})/10 \).

We perform the following steps to obtain uncertainty measures of output growth and inflation:
1. Estimate a VAR model of (14) for the one-quarter ahead forecast at 1983Q4 using only information available to forecasters at the time of submitting their predictions from 1968Q4 to 1983Q4.

2. Obtain residual estimates using (15).

3. For each time period, randomly draw (with replacement) the bootstrap residual $u_i^*$ from the empirical distribution function $F_i$ described above.

4. Generate a new set of bootstrapped data \( \{Z_i, Z_i^{*, k}\} \) and re-estimate the VAR model using the bootstrapped sample.

5. Repeat Steps 3 and 4 two thousand times. The mean value of the $\{\tilde{Z}_i^*\}$ is used as the predicted $Z_i$. The forecast error estimates are produced based on 2000 iterations of the bootstrapping procedure. The standard deviation of the forecast errors from 2000 round of bootstrapping estimation is used as a proxy for uncertainty at the current time period.

6. Repeat Steps 1 to 5 by adding one additional observation. This step is repeated until the end of the sample period.

2.3.4 The Bivariate GARCH Model

The bivariate GARCH model, unlike its univariate counterpart, allows for possible interactions between inflation and output growth in the mean specification. In addition, when compared with the VAR model, the bivariate GARCH model explicitly specifies the observed heteroskedasticity in the conditional variances of inflation and output growth innovations as a GARCH (1,1) process. We estimate the following bivariate GARCH model where the mean specification is a VAR(4) process.
\[ Z_t = \mu + \sum_{i=1}^{4} \Gamma_i Z_{t-i} + \varepsilon_t \]

\[ \varepsilon_t \mid \Omega_{t-1} \sim (0, H_t) \]

\[ H_t = \begin{bmatrix} h_{\Delta y, t} & h_{\Delta y, \pi, t} \\ h_{\Delta y, \pi, t} & h_{\pi, t} \end{bmatrix} \]

where

\[ Z_t = \begin{bmatrix} \Delta y_t \\ \pi_t \end{bmatrix} ; \varepsilon_t = \begin{bmatrix} \varepsilon_{\Delta y, t} \\ \varepsilon_{\pi, t} \end{bmatrix} ; \sqrt{h_t} = \begin{bmatrix} \sqrt{h_{\Delta y, t}} \\ \sqrt{h_{\pi, t}} \end{bmatrix} ; \mu = \begin{bmatrix} \mu_{\Delta y} \\ \mu_{\pi} \end{bmatrix} ; \Gamma_i = \begin{bmatrix} \Gamma_{11}^{(i)} & \Gamma_{12}^{(i)} \\ \Gamma_{21}^{(i)} & \Gamma_{22}^{(i)} \end{bmatrix} \quad \text{and} \quad \Omega_t = \]

\[ \begin{bmatrix} \Delta \end{bmatrix} \]

represents the information set available at period t-1. The conditional variance-covariance process follows the constant conditional correlation model of Bollerslev (1990). The conditional variances of output growth and inflation permit asymmetric responses to shocks of different sign. They are defined in the same way as the univariate GARCH models

\[ h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1} + \gamma \varepsilon_{t-1}^2 \quad \text{and} \quad h_{\Delta y, \pi, t} = \rho \sqrt{h_{\Delta y, t}} \sqrt{h_{\pi, t}} \]

for \( i = \Delta y \) and \( \pi \). The likelihood function of all unknown parameter \( \Theta \) is

\[ l_t(\Theta) = -\frac{1}{2} \log|H_t| - \frac{1}{2} \varepsilon_t^t H_t^{-1} \varepsilon_t. \quad (18) \]

The maximum likelihood estimation method is used to estimate all parameters of the constant conditional correlation model. Just like the univariate GARCH models, we also estimate the symmetric bivariate GARCH model by imposing \( \gamma_i = 0 \) in equation (17).

3. The Data

\[ \text{The use of a constant conditional correlation bivariate GARCH model of inflation and output growth to examine causal relationships between inflation, output growth and their uncertainty is common in the empirical literature (Fountas et al., 2006).} \]
This study employs quarterly data of GDP and GDP deflator obtained from the Survey of Professional Forecasters (SPF). This survey began in 1968 and was conducted by the American Statistical Association and the National Bureau of Economic Research. The Federal Reserve Bank of Philadelphia, however, took over the survey in 1990. An important feature of the SPF which is not commonly observed in other surveys such as the Consensus Economic Survey and the Livingston Survey is that the survey respondents are requested to give (in addition to the usual point forecasts) probabilities for the different intervals of annual average GDP deflator inflation and GDP or GNP growth, starting from 1968Q4 and 1981Q3, respectively. There are open lower and upper intervals, with a number of equal-width intervals between them. The width of the intervals, however, has changed over time; between the period 1981Q3 and 1991Q4 the width was 2%, and outside of this period it was 1%. Although we report the results which include the period when the width interval was changed, our results are robust and remain qualitatively unchanged when compared to a sample which excludes this period.

It is common to use only a particular quarter of the SPF surveys for the benchmark measure of uncertainty. For example, Diebold, Tay and Wallis (1999) and Giordani and Soderlind (2003) consider the first-quarter surveys. The reason for this is that the duration in the definition of the probability forecasts of output growth reported in the SPF does not match with the point forecasts. The point forecasts are for the quarter when the survey is issued and for each of the next four quarters, while the probability distribution refers to the annual change from the previous year to the year of the survey, and from the survey year to the following year. Zarnowitz and Lambros (1987) propose a method of matching the two sets of forecasts. Consider the probability forecast made in Q4 of year \( t-1 \) of the annual change in \( t \) over \( t-1 \). This 4-step probability forecast is matched with a 4-step forecast of output growth by expressing the sum of the point forecasts of the four quarters of year \( t \) as a ratio of the sum of the actual for the first three quarters of year \( t-1 \) and the current forecast of Q4. As for the next survey on Q1 of year \( t \), which gives a 3-step probability forecast of \( t \) over \( t-1 \), a matching 3-step point forecast can be derived accordingly. As is evident, this method results in a matched pair of forecasts of only a single horizon from each survey. Put differently, we only have an annual series of 1-step forecasts if we
consider the surveys made in the third quarters of each year, or an annual series of 2-step forecasts if the second quarter surveys are employed. The difference in forecast horizon for each quarter of the survey implies that we are not able to utilise all quarters of the SPF data. Following this convention, we employ SPF data for the first quarter of each year. Given that quarterly comparable data for real GDP growth are only available from 1981Q3, our analysis consider a sample period from 1982Q1 to 2008Q4.

Empirical measures of macroeconomic uncertainty are obtained from utilising the quarterly real-time and revised data maintained by the Federal Reserve Bank of Philadelphia. Inflation is computed as

\[ \pi_{t,j} = \left( \frac{1}{n} \sum_{j=Q1}^{Q4} I_{t,j} / \frac{1}{4} \sum_{i=Q1}^{Q4} I_{t-4,i} \right)^{-1} \times 100 \]

where \( I_t \) is the chain-weighted GDP price index and \( n = 1, 2, 3 \) and 4 for Q1 to Q4 respectively. Output growth is computed in the same way as inflation except that real GNP is employed. The last vintage of real-time data in 2008Q4 constitutes the revised data as this data set would have undergone a significant amount of revision over time. For both data sets, our sample starts from 1966Q4.

4. Empirical Results

4.1 Is there a best practice for constructing empirical measure of macroeconomic uncertainty?

Tables 1 and 2 show correlations of empirical measures of uncertainty with the benchmark survey measure. Specifically, Table 1 reports the correlation results for a class of GARCH models, while Table 2 documents results for the VAR and SVAR models. It can be seen that, by and large, all of the correlations between the time-series measure and the survey measure are significantly different from zero at the 5% level. Uncertainty measures for real output growth and inflation that are generated by bivariate GARCH, VAR, SVAR and bootstrapped VAR approach tend to be highly significant even at the 1% level. Although the correlation is able to determine the statistical significance of the association between the two measures of uncertainty, it does not show the extent by which the empirical measure tracks movements of the
survey measure. We plot the empirical and benchmark measures together to examine the extent by which they co-move together, and whether the empirical measure over or under estimates the benchmark measure of uncertainty. Figures 3 and 4 provide a summary of inflation uncertainty plots produced by the two classes of GARCH and VAR models, respectively. The corresponding output growth uncertainty plots are shown in Figures 5 and 6.

- Tables 1 and 2 about here -

When comparing correlations produced by the fixed-sample and increasing-sample recursive methods, we find that proxies produced by the fixed-sample recursive method tend to be more highly correlated with the survey measure. The result is robust to the types of data and models used. For example, for the class of univariate and bivariate GARCH models, we find inflation uncertainty proxies produced by employing the fixed-sample recursive method and real-time data showcase a significant improvement in their correlations with the survey measure compared to the increasing-sample recursive method. The increment in the correlation is as large as 0.18 in the bivariate asymmetric GARCH model. The same pattern is also observed in revised data, except for the class of univariate GARCH models.

In addition to the apparent improvement in the uncertainty proxy correlation, there is further evidence that the fixed-sample recursive method yields inflation uncertainty proxy that better matches movements of the survey measure. This is particularly true in GARCH models. Referring to columns 1 and 2 in Figure 3, it can be seen that in the class of univariate GARCH models and for the sample starting period examined, the increasing-sample recursive method yields a proxy that peaks in 1983Q1. This steep increase in the inflation uncertainty proxy is likely to arise from data revision that takes place in recent vintages of real-time data. Given that the increasing-sample recursive method retains observations in past periods of the real-time data sample, it is likely that some of the data would have been subjected to significant revisions and this would have influenced the estimates of inflation uncertainty proxy. These results suggest that the fixed-sample recursive method is a preferred approach to deriving empirical measure of uncertainty.
Another noteworthy observation is that the use of real-time data is likely to yield empirical measure of uncertainty that is more highly correlated with the survey measure than revised data. This result which is evident in the correlation Tables 1 and 2 is, in general, robust to the estimation method and models employed. Figure 3 further shows that the use of revised data yields empirical measure of inflation uncertainty which is higher than the survey measure at the starting and some other periods of the sample.\(^9\) When interpreted together with the results for the fixed-sample recursive method, they suggest that the use of real-time data with fixed-sample recursive method is a good alternative to the current practice of employing a non-recursive method and revised data in constructing empirical measure of inflation uncertainty. In addition, it is important to recognise that the application of a non-recursive method to revised data produces an \textit{ex post} measure of uncertainty, which may not be realistic in practice. This is because forecasters do not have access to revised data let alone exploit the information content to predict the level of uncertainty about future inflation and output growth. On the other hand, the use of real-time data with fixed-sample recursive method is consistent with the \textit{ex ante} definition of uncertainty measure and the proxy is, in principle, comparable with the survey measure.

A quick comparison between Figures 3 and 4 indicate that the GARCH and the bootstrapped VAR approach produce inflation uncertainty estimates that track the survey measure better than proxies generated by the VAR and SVAR models. The VAR and SVAR models produce very smooth inflation uncertainty estimates and fail to capture the movements of the survey measure. This smooth feature stems from the homoskedastic assumption of the model and the use of equal weights for all residuals.

\(^9\) The high inflation uncertainty estimates at the starting period of the sample can be rationalised by the relatively high inflation level during which the Volcker’s monetary policy experiment of the late 1970s was implemented. The Fed adopted a new set of operating procedures at that time that featured increased emphasis on a particular measure of bank reserves and reduced emphasis on short-term interest rates to curtail the high level of inflation. The monetarist experiment was successful and lowered inflation from around 11 or 12 percent (per year) to a magnitude in the vicinity of 4 or 5 percent. Given that the GARCH inflation uncertainty estimate for 1982 is generated by data from the last 15 years, it is not surprising to find the increasing-sample recursive method with real-time data and the use of revised data yield a significantly higher conditional variance estimate for inflation.
when constructing the standard error of the forecast. Although the correlation results for VAR and SVAR models are higher than those of GARCH and bootstrapped VAR models, the plots in Figures 3 and 4 speak to their failure to accurately track movements of the survey measure.

- Figures 5 and 6 about here -

Figures 5 and 6 show plots of output growth uncertainty proxy from the two classes of GARCH and VAR models. It is obvious that, contrary to the plots of empirical measure for inflation uncertainty, output growth uncertainty estimates are higher than the survey measure. The GARCH models, which are shown to produce inflation uncertainty proxies that follow movements of the survey measure well, fail to produce as good a measure for growth uncertainty. Of the different empirical proxies that are examined, the bootstrapped VAR measure minimises the gap between the level of empirical measure and the survey measure and therefore tracks the survey measure better than the other proxies. The fact that the GARCH and VAR proxies for growth uncertainty are higher than the survey measure it cautions the use of such models even though they have featured extensively in the empirical literature. There is further evidence that the Great Moderation, where the volatility of output growth has started to decline in the mid 1980s, is captured in many of the plots by the downward trend observed in output growth uncertainty proxies.

4.2 Does the multivariate approach improve the performance of univariate models of uncertainty?

Table 1 shows that the correlations of inflation uncertainty based on a bivariate GARCH model are higher than their univariate GARCH counterparts. It is also evident from the plots of GARCH models in Figure 3 that the bivariate GARCH models yields superior inflation uncertainty estimates that follow movements of the survey measure better than the univariate model. In particular, some of the spikes that appear in the univariate GARCH estimates are no longer observed in the bivariate GARCH proxy. Of the different GARCH models, data and estimation methods considered, we find that the bivariate asymmetric GARCH model with fixed-sample
recursive method and real-time data produces the best proxy of inflation uncertainty and it matches the survey measure very well. This is documented in the bottom most left hand corner of Figure 3. In contrast, we fail to find any improvement in the proxy for output growth uncertainty for the bivariate GARCH model. This is evident both from the correlation results in Table 1 and Figure 5.

Turning to the class of VAR models, we find evidence in support of a structural model that accounts for the effects of monetary policy on inflation and output growth. In the 4-variable VAR model, there is evidence that the inflation uncertainty proxy captures movements of the survey measure (see Figure 4); there is a rise in uncertainty at the start of the sample followed by a gradual fall in the level towards the middle of the sample before flattening out in the second half of the sample. Such a feature of inflation uncertainty proxy is missing in the 2-variable VAR model. Likewise, the 4-variable VAR model also generates a proxy for growth uncertainty that captures movements of the survey measure (see Figure 6). In particular, the proxy captures the downward trend that is observed in the survey measure. Taken together, these results indicate that an informatively richer model like the 4-variable VAR specification could deliver a more meaningful uncertainty proxy.

In column two of Figure 6, the SVAR model provides only marginal improvements over the proxies of the 4-variable VAR model. However, in this model the use of fixed-sample recursive method yields superior uncertainty estimates compared to the increasing-sample recursive method; the fixed-sample recursive method generates proxies for both inflation and growth uncertainty that capture the rise and fall in the survey measure. In addition, we find that the proxies are not sensitive to the type of data used, be it revised or real-time data, although the choice of the methodology matters. Between the SVAR and the 4-variable VAR model, we recommend the former model. This is because the SVAR model imposes restrictions which are justified by theory to uncover the structural shocks so that appropriate economic interpretations can be given to these shocks. On the contrary, the VAR model suffers from the problem of identification. With no restrictions imposed on the VAR models, the contemporaneous correlation of the shocks makes it difficult to interpret the shocks.
4.3 Are there important features in time-series models that must be accounted for to improve their uncertainty forecasts performance?

As noted previously, GARCH models tend to produce inflation uncertainty estimates that are more consistent with the survey measure. This stems from the fact that such models explicitly account for the heteroskedasticity observed in inflation and output growth innovations. Similarly, the bootstrapped VAR approach yields inflation uncertainty proxy that squares well with the survey measure. It can be inferred from these results that a model which accounts for heteroskedasticity in the innovations of inflation and output growth will generally outperform a model that does not take into account of the heteroskedastic residuals.

Within the class of univariate GARCH models, we find that both the asymmetry and levels dependence in the conditional variance of inflation are important features that will improve the proxy performance. This can be seen from the correlations in Table 1 in which we find that an asymmetric GARCH model produces higher correlations than a symmetric GARCH model. This result is robust whether we consider a univariate or a bivariate GARCH model. In the class of univariate GARCH models for inflation uncertainty, we further find that models which accommodate for levels dependence and asymmetry in the conditional variance of inflation, equally, yield as good a correlation as, if not better than, the symmetric GARCH models. Levels dependence in the conditional variance of inflation is a feature that is consistent with the Friedman’s (1977) hypothesis, that is inflation uncertainty peaks with the level of inflation.

Finally, we consider whether a time-varying parameter model with GARCH errors produces uncertainty measure that is superior to a standard GARCH model. The correlation for the time-varying GARCH model of inflation is 0.51 which is marginally higher than the proxy produced by a GARCH model with revised data and the non-recursive method. The marginal improvement in the correlation could be due to the fact that the sample period examined comprises largely of low inflation data with little variation. Moreover, after the monetarist experiment of the late 1970s and early 1980s, inflation level has fallen to record low. As a result, fitting a time-varying
GARCH model produces little gain relative to a standard GARCH model. As for growth uncertainty, the correlation for the time-varying model drops rather drastically suggesting that the use of a time-varying GARCH model for output growth uncertainty may be inappropriate.

5. Conclusion

In the attempts to derive measures of macroeconomic uncertainty, the lack of direct observations on uncertainty about future inflation and output growth has led researchers to construct various uncertainty proxies. When faced with a growing spectrum of empirical models for constructing inflation and output growth uncertainty proxies, they often encounter the dilemma of choosing the most appropriate model and employing the right proxies that are consistent with the survey measure in empirical research. This paper addresses these practical issues by considering a plethora of time-series models comprising VAR, SVAR, univariate and bivariate GARCH, and time-varying GARCH models. When compared with the benchmark proxy, which is the uncertainty measure reported by individual forecasters in the Survey of Professional Forecasters (SPF) for the period 1982-2008, we find that there are noteworthy criteria that a researcher should satisfy in order to ensure that the generated time-series proxy closely replicates the survey measure. The use of time-series uncertainty proxies which deviate significantly from the survey measure could yield empirical results that are suspect.

Our findings can be summarised as follows. First, the results suggest models that account for heteroskedastic errors in inflation and output growth produce uncertainty proxies that track the behaviour of the survey measure well. Of the two models that accommodate heteroskedasticity in inflation and growth errors, evidence from graphical plots of these measures and their correlations indicate that the performance of the uncertainty proxy produced by a bivariate GARCH model is superior to a bootstrapped VAR approach. On the other hand, while VAR models are known to provide good short-term forecasts for macroeconomic variables like output growth and inflation, we fail to find evidence in favour of their application to generate measures of uncertainty. The SVAR models, likewise, produce only marginal improvement in their uncertainty proxy compared with the 4-variable VAR
counterpart. Nonetheless, we find that an informatively richer model such as one which accounts for the effects of monetary policy on inflation and output growth yields better uncertainty proxies that track movements of the benchmark measure. Secondly, when adopting a GARCH model to derive uncertainty proxies, there is evidence to suggest that an asymmetric conditional variance specification, which identifies the effect of a differently signed shock on uncertainty, improves upon the performance of a proxy produced by a symmetric GARCH model. Thirdly, the use of real-time data when combined with a fixed-sample recursive method also gives rise to an uncertainty proxy which is not only consistent with the definition of an ex ante measure, but also surpasses in its performance to a proxy that is generated by using revised data and a non-recursive method. This result, therefore, challenges the current practice of using revised data and adopting a non-recursive approach in estimating macroeconomic uncertainty. It is also possible that the conventional approach in generating uncertainty proxy is liable to lead to errors in inferences when used in empirical research. Fourthly, we also find that the fixed-sample recursive method produces uncertainty proxies that are superior to the time-varying parameter model, and therefore imply that the use of the latter model may deliver little advantage in practice, unless there are significant variation in the level of inflation and output growth. Finally, while the models examined in this paper mostly produce inflation uncertainty estimates that are comparable to the survey measure, they tend to over estimate the level of uncertainty for output growth. For this reason, future research should consider alternative methods for constructing a time-series measure of output growth uncertainty.
References


<table>
<thead>
<tr>
<th>Table 1: Correlations of GARCH Uncertainty Measures with $E(\sigma_i)$ for 1982 – 2008</th>
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<tbody>
<tr>
<td><strong>Inflation</strong></td>
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<tr>
<td><strong>Univariate</strong></td>
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<td><strong>Real-time data</strong></td>
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<td>Increasing-sample recursive method</td>
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<td>AGARCH</td>
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<tr>
<td>AGARCHL</td>
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<td>Fixed-sample recursive method</td>
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<tr>
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<tr>
<td>AGARCHL</td>
</tr>
<tr>
<td><strong>Revised data</strong></td>
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<td>Fixed-sample recursive method</td>
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<td>AGARCHL</td>
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<tr>
<td>Non-recursive method</td>
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<tr>
<td>AGARCHL</td>
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<tr>
<td>Time-varying</td>
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<tr>
<td>GARCH</td>
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</tbody>
</table>

Note: For both classes of univariate and multivariate GARCH models the prefix ‘A’ denotes Asymmetric. The suffix ‘L’ for univariate GARCH models denotes levels dependence in the conditional variance. The critical values for 25 degrees of freedom and at the 5% and 1% significance levels are 0.381 and 0.487, respectively.
Table 2: Correlations of VAR, SVAR and Bootstrapped VAR Uncertainty Measures with $E(\sigma_i)$ for 1982 – 2008

<table>
<thead>
<tr>
<th></th>
<th>Inflation</th>
<th>Real-time data</th>
<th>Output Growth</th>
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<tr>
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<td>Increasing-sample</td>
<td>Fixed-sample</td>
<td>Increasing-sample</td>
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<tr>
<td>Bivariate VAR</td>
<td>0.74</td>
<td>Bivariate VAR 0.82</td>
<td>Bivariate VAR 0.77</td>
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<tr>
<td>4-variable VAR</td>
<td>0.72</td>
<td>4-variable VAR 0.79</td>
<td>4-variable VAR 0.61</td>
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<tr>
<td>SVAR</td>
<td>0.74</td>
<td>SVAR 0.79</td>
<td>SVAR 0.45</td>
</tr>
<tr>
<td>Bivariate</td>
<td>0.56</td>
<td>Bivariate      0.66</td>
<td>Bivariate 0.63</td>
</tr>
<tr>
<td>BOOTVAR</td>
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|                | Increasing-sample | Fixed-sample | Increasing-sample | Fixed-sample |
| Bivariate VAR  | 0.70      | Bivariate VAR 0.78 | Bivariate VAR 0.51 | Bivariate VAR 0.72 |
| 4-variable VAR | 0.69      | 4-variable VAR 0.77 | 4-variable VAR 0.67 | 4-variable VAR 0.73 |
| SVAR           | 0.73      | SVAR 0.80        | SVAR 0.60        | SVAR 0.75       |
| Bivariate      | 0.57      | Bivariate 0.69    | Bivariate 0.70   | Bivariate 0.77  |
| BOOTVAR        |           | BOOTVAR         |                | BOOTVAR         |

Note: The bivariate VAR and bootstrapped VAR (BOOTVAR) models are a system of real output growth and inflation. The four variables VAR and SVAR are made up of real output growth, inflation, Federal funds rate and a long-term interest rate. The critical values for 25 degrees of freedom and at the 5% and 1% significance levels are 0.381 and 0.487, respectively.
Figure 1: Aggregate probability density inflation forecast
Figure 2: Aggregate probability density output growth forecast
Note: AGARCH denotes asymmetric GARCH model, AGARCHL denotes asymmetric GARCH model with levels effect and BiGARCH denotes Bivariate GARCH model. Column 1 uses fixed-sample (FS) recursive method with real-time (RT) data while column 2 uses increasing-sample (IS) recursive method. Columns 3 and 4 use revised data (RD) while column 5 uses non-recursive (NR) method to estimate uncertainty. The solid line represents the benchmark survey measure $E(\sigma)$. 

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Figure 4: Inflation uncertainty from VAR, SVAR and time-varying GARCH models

Note: The 2-VAR and 4-VAR models denote the two-variable and four-variable VAR models, respectively. The TV-GARCH model denotes the time-varying parameter GARCH model. BOOTVAR refers to the bivariate bootstrapped VAR model.
Figure 5: Output growth uncertainty from GARCH models

Note: See note to Figure 3.
Figure 6: Output growth uncertainty from VAR, SVAR and time-varying GARCH models

Note: See note to Figure 4.