Communication Breakdown: consultation or delegation from experts with uncertain bias

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Abstract

When communicating with an uninformed decision maker, the motives behind an expert’s message are often unclear. To explore this situation and investigate its impact on organizational design we extend the cheap-talk model of Crawford and Sobel (1982) to allow for uncertainty over the expert’s bias. We find that, in contrast to Dessein (2002), it is possible that the decision maker prefers communication to delegation; that is, it can be optimal for a decision maker to retain control and to solicit advice from the expert.

Key words: delegation, communication, uncertainty, bias, cheap talk.

JEL classifications: D23, D83, L23.

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1 Introduction

Having the relevant specific knowledge is critical for effective making (Jensen and Meckling (1995)). However, those responsible for this task often lack key information. A decision maker might be uninformed because of the highly specialized knowledge needed to choose between potential projects. In such situations, the uninformed decision maker may seek the advice of an informed expert about what actions she should take – for example, the senior manager of a plant might ask a shop-floor manager advice on a new project or a CEO of a software firm could solicit a recommendation of its research department regarding the development of a new product.

In these situations, more often than not, the project selected affects the welfare of both the principal and the agent. For instance, a middle manager’s division is likely to be affected by a CEO’s decision; similarly, shareholders often rely on the advice of financial analysts, who in turn may have a vested interest in the prevailing stock price. Where the preferences of the agent and the principal do not coincide, the agent will have an incentive to behave strategically by distorting or obfuscating the truth in any message they communicate. This paper analyzes the strategic communication process between an expert advisor and an uninformed principal and its impact on an organization’s decision making and communication protocols.

Crawford and Sobel (1982) (hereafter CS) analyzed strategic communication in a cheap-talk game between a perfectly-informed sender (or agent) and a receiver (principal). The sender observes the state, which takes the value of a random variable, before sending a message to the receiver. Upon observing the message, the receiver takes an action. The state and the receiver’s action determine the payoffs for both players. The preference divergence, or the sender’s bias, is captured by a constant parameter $b$, which is known by both agents.

When the preferences of the two agents differ, CS found that regardless of the size of the bias, there cannot be an equilibrium in which the sender reveals the true state. Instead, the equilibrium involves a partition of all of the states of nature and
the agent’s message only identifies a partition element that includes the true state of nature. As the sender’s noisy signal is credible, the receiver in turn chooses the action that maximizes his expected utility given his (correct) probabilistic belief of the distribution of the state. CS show that more informative equilibria are preferred by the receiver, as communication is noisy, and that the receiver’s welfare is lower than if he were perfectly informed.

As an alternative that avoids this strategic communication, the decision maker in an organization could delegate his decision-making rights to the expert. Extending Crawford and Sobel (1982), Dessein (2002) found that with standard assumptions over the decision maker’s best actions, delegation is optimal ‘whenever informative communication is possible’ (p.822). In general, the loss of information in communication leads to a greater reduction in welfare for the receiver than the loss of control under delegation, especially when biases are small (Dessein, 2002, p812).

In reality, however, the choice between communication and delegation is often much more complex. The experience of Nestlé highlights this point. Nestlé initially decided to delegate more decisions; this strategy, however, was not successful and the company decided to revert to centralized decision making, relying on the transfer of the requisite information to these key decision makers (The Economist, 5 August 2004). Similarly, the marketing divisions of General Motors and Dell Computers have made conflicting choices:

General Motors announced this summer that it will merge its 5 marketing divisions into one [communication], ... Meanwhile, Dell Computer actively decentralizes [delegates] its marketing by assigning fewer market segments to divisions as they grow. Dell has 12 marketing divisions now, compared with 4 in 1994. (Donath, 1998, p.9)

One of the key assumptions of CS and Dessein (2002) is that the preference divergence between the two parties is common knowledge; that is, while the decision maker is uninformed about the appropriate project to implement for the realized state of the world, he has perfect knowledge about the expert’s bias. In the language of
Dessein (2002), there is a systematic and predictable difference between the preferences of the principal and the agent. In many contexts, however, the decision maker will be unsure of the underlying objectives of the expert – that is, the bias of the agent is not systematic and predictable. Again, examples abound: a political leader may be uncertain about the political leaning of an advisor; the CEO might be unsure whether the manager is an empire builder or whether he is effort averse; and only some financial analysts wish to short sell a stock.

We model the situation when: the principal is uninformed; the principal is uncertain regarding the bias of the expert; and the action taken affects the utility of both the principal and the agent. To this end we extend CS to allow for the receiver to be uncertain about the sender’s bias; specifically, we allow the sender’s bias, which is private information, to take on two possible values. We also assume, partly to reflect the potential coarseness and imprecise nature of communication in real organizations, that only two possible messages can be sent by the sender (either Low or High).

By allowing for uncertainty with respect to the sender’s bias, we increase the range of biases for which communicative equilibria are possible. In order to further investigate these equilibria, we define two different types of expert/sender: (1) an informative sender, who in equilibrium is willing to send both types of messages depending on the realized state of the world; and (2) an uninformative sender, who only sends one message across all states of the world. Further to that, even in situations when the sender will always be informative, a sender with a small – or moderate – bias compared to the principal will behave differently to an informative sender with a larger – or extreme – bias, affecting the relative performance of communication to delegation.¹

We find that if the two biases have the same sign, the result of Dessein (2002) that delegation is always preferred to communication holds. This is not always the case, however. For example, if the seller’s potential biases have opposite signs, it is possible that the principal prefers communication over delegation. The intuition for this

¹These two terms, moderate and extreme, are defined precisely in Section 3.
result is that uncertainty over the expert’s preferences can mute the strategic effects of communication, encouraging a biased expert to send more informative messages. The reduction in the loss of information can be sufficient to allow communication to dominate delegation. The model also shows that if both types of experts are extreme, it is better to communicate. Moreover, in the case in which one of the types of expert is moderate and one is extreme, there is a threshold probability (for the type being extreme) above which the principal will prefer to communicate (and maintain centralized decision making). Intuitively, by retaining control rights under communication the decision maker can maintain incentives for information transmission from unbiased experts, whilst insuring against (very biased) experts that might want to implement a project not in his interest.

Several other papers have also addressed the communication-or-delegation question. Ivanov (2008) showed that if the receiver can ‘optimally’ restrict the sender’s information, communication always dominates delegation. Krahmer (2006) compared communication and delegation when utility is transferable between the sender and receiver and contracts are only partially incomplete. We take a different but complementary approach. Rather than allowing the receiver to have some control on communication, we focus on the effect of introducing greater uncertainty in terms of the sender’s bias.

In another related study, Blume et al. (2007) examined information transmission when, with positive probability, the message sent is misinterpreted by the receiver and, as a result, is uninformative. Interestingly, they find that adding some noise to the sender’s signal can almost always improve the welfare of all parties as the noise creates incentives for the sender to reveal more information, and the value of this additional information outweighs that utility loss from misinterpretation.

Other papers have focussed on the bias of the sender. For example, Hughes and Sankar (2006) found that the partition equilibrium in the CS model is robust to uncertainty with respect to the sender’s preferences. Specifically, they showed the existence of a partitional equilibrium when the manager (sender) is biased, which arises in their model due to differences in the manager’s aversion to litigation and
the investor’s (or receiver’s) exposure given their level of insurance and that this partitional equilibrium is robust to investor uncertainty with respect to the manager/sender’s bias (the sender’s bias can take one of two values). Our focus here is not on the existence of partitional equilibrium, but rather the relative advantages of communication over delegation. Dimitrakas and Sarafidis (2005) generalized the CS model to allow for uncertainty with respect to a sender’s bias and they found that all equilibria are partitional equilibria. Moreover, in a similar way, Dimitrakas and Sarafidis (2005) found that a partitional equilibrium with a very limited of partitions (for example 2 partitions) comes very close to the limit equilibrium, suggesting little information is lost when the sender is restricted to a small number of messages, as in the model here. These results allows us to simplify our analysis by focusing on a 2 message model, which is akin to a 2-partition equilibrium. Using a similar set-up to our model, Li and Madarasz (2008) examined whether, prior to communication, disclosure by the expert of their bias should be mandatory. They find, as shown by CS, that all communicative equilibria are partitional once the expert’s bias is known, meaning that communication inherently involves a loss of information. Moreover, communication without mandatory reporting (so that there is uncertainty regarding the sender’s bias) can make communication itself more informative, improving the welfare of both the expert and the decision maker. While the design of their model is similar to ours, Li and Madarasz (2008) do not address the impact of uncertainty over the expert’s preferences and organizational structure.

Morgan and Stocken (2003) analyzed communication when an investor/receiver is uncertain as to whether the stock analyst (sender) is unbiased (has a bias equal to zero) or has a positive bias, in that she wishes to inflate the value of the stock. In equilibrium, uncertainty about the sender’s incentives means that it is not possible to credibly reveal good news about the valuation of the stock, even if the sender is unbiased. They also show that institutional restrictions on the communication process, in that the analyst can only send a broad messages that rank the stocks (for example, sell, hold or buy) arise endogenously in equilibrium.

In a paper closely related to ours, Ottaviani (2000) constructed a model where the
sender’s bias is symmetric. The sender’s bias can take a positive or negative value with equal probability ($b$ or $-b$) — capturing situations where preference divergences are random rather than systematic. He found that the receiver’s welfare is always higher under communication than it is under delegation. In an important departure from Ottaviani (2000) we take a more general approach in that we do not require that the biases are symmetric.

2 The Model

Consider, in turn, the model under: (a) communication, when the principal opts to retain her decision-making rights and consults an informed agent or expert; and (b) delegation, when the principal relinquishes her decision-making rights regarding the project selection to the agent.

Communication

Here, the underlying problem is that an uninformed decision maker must choose a project. Given she is uninformed, the decision maker can elicit a costless message from an informed expert. However, this expert is biased in that his preferences over these projects differ to the preferences of the decision maker.

Consequently, in the model there are two players, a sender and a receiver. The sender observes the state $s$ that is a random variable drawn from the interval $S = [0, 1]$. The distribution of the state is assumed to be uniform. The sender also observes her bias, $b$, that may take two values, $b_1$ or $b_2$ where $-b_1 \leq b_2 \leq b_1$. There is a probability of $p$ $(1 - p)$ that the sender’s bias is $b_1$ $(b_2)$. The prior distributions of the sender’s bias and the state are common knowledge to both sender and receiver. In this paper, we will refer to the sender’s bias as her type. Note that as there are two types of uncertainty in the model (over the state and the sender’s bias) this labeling is made for the purpose of clarity.\(^2\)

\(^2\)It is also worth mentioning that the majority of the literature uses the term ‘type’ to describe the sender’s observation of the state.
The timing of the game is as follows. The sender, of type $b_i$ where $i = 1, 2$, observes $s$, and sends a message, $m \in M$, to the receiver. After receiving $m$, the receiver chooses an action $a$ that determines the utilities of both players. For simplicity, the sender’s utility from action $a$ is:

$$U^S = -|a - (s + b)| \quad (1)$$

while the receiver’s utility from action $a$ is:

$$U^R = -|a - s|. \quad (2)$$

Note that here, for ease of exposition, we depart from many of the applications of the CS mode in that we assume linear utilities; for example, the leading example of CS (p.1440) uses a quadratic utility. This departure is not critical as it is the difference between a party’s preferred action and the implemented action that matters.

The following standard assumptions are made: sending a message is costless for the sender; the message cannot be verified by the receiver; and the receiver also cannot commit to a decision rule *ex ante*. We also make one additional assumption. Upon observing the state, the sender can send one of only two messages - *Low* or *High*, specifically $M = \{\text{Low, High}\}$. That is, while the state of nature and the number of actions that can be taken is infinite, the number of reports that can be sent is restricted. There are several reasons as to why the message space might be restricted. As noted above, Morgan and Stocken (2003) found that a sender may wish to implement a restriction on the possible messages that can be sent to simple ranking (*Low* or *High* for example). This result arises endogenously in their model and, furthermore, does not necessarily result in any loss if information. Moreover, even when the message space is unrestricted in the original CS model, there is an equilibrium in which only two distinct messages are sent in equilibrium — a two-partition model — that is equivalent to the equilibrium in our model when $b_1 = b_2$.\(^3\)

\(^3\)We utilize this equivalence to compare the results from this model to the two-partition CS model.
Another reasons for our binary message space could be that the communication technology is coarse. It is often not possible to precisely describe exactly the type of project required, particularly as we are assuming a costless communication process. Dwyer (1999, cited in Joiner et al., 2002) argued that if the sender must resort to technical language to be more precise, this might actually reduce the clarity of the message. Others have also made a similar assumption: for example Takáts (2007) assumes that the sender cannot report all her information. This coarseness could arise from the sender’s inability to precisely articulate or from the receiver’s lack of expertise of comprehension, leading the receiver to understand the rough jist of the message, while being cloudy on the detail. However, it might be that case that the sender can communicate in broad terms about the project she recommends, be it ‘small’, ‘large’, ‘locate in region X’, etc.\footnote{One could argue that the uninformed receiver could expend effort to understand the message, but this would be equivalent to investing effort into becoming informed, as in Aghion and Tirole (1997), possibly eliminating the need to communicate with an expert. Here we follow CS and Dessein (2002) in assuming the information structure as exogenous.}

The solution concept we shall employ is the Bayesian Nash Equilibrium. The decision maker’s beliefs, $P(\cdot|\text{Low})$ and $P(\cdot|\text{High})$, be formed using Bayes’ rule for possible messages $\text{Low}$ and $\text{High}$. The decision maker’s actions, $X_L$ and $X_H$, maximize his expected utility given his beliefs $P(\cdot|\text{Low})$ and $P(\cdot|\text{High})$. The expert’s messages, $\text{Low}$ or $\text{High}$, maximize her utility for all $s$ given the decision maker’s strategy.

Our model satisfies the properties of Dimitrakas and Sarafidis (2005), who found that all equilibria of the model are partitional equilibria. Consequently, we do not fully characterize the equilibria of our model. Rather, as our purpose in this paper is to generate a counter example to the delegation result of Dessein (2002), we focus on the simple 2-message equilibrium that is equivalent to a two-partition equilibrium. This means that we are comparing the optimality of delegation to communication in the least informative (and least advantageous) communicative equilibrium, suggesting any dominance of communication would only be enhanced with more refined communication.
Delegation

The model of delegation is as follows. Before the state of nature is revealed, the receiver delegates control of the action to the sender. Upon observing the state of nature, the sender chooses an action that determines the utility of both players.

Utility functions for sender and receiver are unchanged under delegation. As the sender’s signal is perfect, it is assumed she can choose any action with complete precision under delegation. Thus, the sender will choose her optimal action, $s + b$, yielding utilities of $U^S = 0$ and $U^R = -b$ for the sender and receiver respectfully.

Model in context

It is worth placing our model in the context of the previous literature on strategic communication and the optimality of delegation. CS showed that if there is certainty over sender’s bias – which we termed the benchmark case where $b_1 = b_2$ – when the state is distributed uniformly communication is informative for $|b| < \frac{1}{4}$. Following from this, Dessein (2002) showed that when preferences are linear-quadratic the receiver optimally delegates to the sender whenever communication is informative. In the case where $b_1 = -b_2$, and $p = 0.5$, Ottaviani (2000) showed that communication is informative when $|b| < \frac{1}{2}$ (hereafter, the symmetric case) and that when preferences are linear-quadratic, the receiver always prefers communication over delegation. While using a different utility function, Morgan and Stocken (2003) also allow for some uncertainty with respect to the sender’s bias. In their model $b_1 = 0$ and $b_2 = b$. Here we relax the assumptions of Dessein (2002), Ottaviani (2000) and Morgan and Stocken (2003) by modelling communication for arbitrary $b_1$, $b_2$ and $p$. That is, we provide a complete characterization of the parameter values for which informative communication is possible when the receiver is uncertain of the sender’s bias.
3 Solving the model

Upon receiving the report \( \text{Low} (\text{High}) \), the receiver chooses the action \( X_L (X_H) \) to maximise his utility. Without loss of generality, \( X_L < X_H \). A sender of type \( b_i \) reports \( \text{Low} \) if \( U_i^S(X_L) \geq U_i^S(X_H) \) and reports \( \text{High} \) if \( U_i^S(X_H) > U_i^S(X_L) \), where \( i = 1, 2 \). As noted there are many equilibria in this game. We focus on equilibria with the feature that if a sender with a bias \( b' \) sends \( \text{Low} \) in equilibrium, every sender with a bias \( b < b' \) sends \( \text{Low} \) in equilibrium; and if sender with bias \( b' \) sends \( \text{High} \) in equilibrium, every type of sender with \( b > b' \) will also send message \( \text{High} \) in equilibrium. This allows us to focus on the sender of bias \( b \) that is indifferent between sending either message.

The state where Type \( b_i \) is indifferent between reporting \( \text{Low} \) and \( \text{High} \) is \( T_i \). Type \( i \)'s indifference implies that at state \( T_i \)

\[
(T_i + b_i) - X_L = X_H - (T_i + b_i)
\]

where the payoff to type \( b_i \) sending message \( \text{Low} \) or sending message \( \text{High} \) are displayed on the left-hand side and the right-hand side, respectively. Solving for \( T_i \):

\[
T_i = \frac{1}{2}(X_L + X_H) - b_i
\]  

Equation (3) implies that Type \( i \) reports \( \text{Low} \) for all states less than \( T_i \), and \( \text{High} \) for all states greater than \( T_i \). Given that \( b_1 > b_2 \), \( T_1 < T_2 \). Intuitively, Type 1’s higher bias implies that she prefers the higher action at a lower state of nature.

The following definitions are introduced to describe the reporting behavior of the two types:

**Definition 1.** Informative - A sender type is informative if she reports, depending on the state of the world, both \( \text{Low} \) and \( \text{High} \) in equilibrium. That is, her indifference point lies between 0 and 1.

**Definition 2.** Uninformative - A sender type is uninformative if in equilibrium she
only sends one message across all states - that is, her reports are uninformative.

Two further definitions are introduced to describe the reporting behavior of informative types. These will be useful in organizing and explaining the results.

**Definition 3. Moderate** - An informative type is classed as **moderate** if her indifference point lies between $X_L$ and $X_H$. Intuitively, if a sender’s bias is small than her indifference point is close to the midpoint of $X_L$ and $X_H$.

**Definition 4. Extreme** - An informative type is **extreme** if she reports both *Low* and *High* in equilibrium, but reports *High* when $s = X_L$, or *Low* when $s = X_H$. As a type’s bias increases, her indifference point moves further from the midpoint of $X_L$ and $X_H$.

Obviously, communication is only possible when at least one type is informative. For $T_1 < T_2$, at least one of the following conditions must hold:

1. Both types are informative, i.e. $0 < T_1 < T_2 < 1$;
2. Type 1 is uninformative and Type 2 is informative, i.e. $T_1 < 0 < T_2 < 1$;
3. Both types are uninformative, that is no effective communication is feasible.

We investigate equilibria in each of the following conditions in turn.

### 3.1 Both types informative

There are 3 potential subcases where both types are informative.

#### 3.1.1 Subcase 1 — $b_1$ and $b_2$ moderate

If $b_1$ and $b_2$ are close to zero, then $T_1$ and $T_2$ are close to the midpoint of $X_L$ and $X_H$. Both players are moderate when $0 < X_L < T_1 < T_2 < X_H < 1$. The sender’s problem is shown in Figure 1. As preferences are linear and because the receiver takes $T_1$ and $T_2$ as given, he solves the following problem upon observing a *Low* signal:

$$\min_{X_L} \frac{p}{2} [X_L^2 + (T_1 - X_L)^2] + \frac{1-p}{2} [X_L^2 + (T_2 - X_L)^2].$$

(4)
That is, if the receiver observes a Low signal, with probability \( p \) the state is between 0 and \( T_1 \) and with probability \( 1 - p \) the state is between 0 and \( T_2 \). The receiver then chooses \( X_L \) to minimise his welfare loss, given the distribution of states in which Low may be sent.

The receiver’s problem upon receiving High is:

\[
\min_{X_H} \frac{p}{2} [(X_H - T_1)^2 + (1 - X_H)^2] + \frac{1-p}{2} [(X_H - T_2)^2 + (1 - X_H)^2].
\]  
(5)

If the receiver is sent a High signal, with probability \( p \) the state is between \( T_1 \) and 1 and with probability \( 1 - p \) the state is between \( T_2 \) and 1. Similarly, the receiver chooses \( X_H \) to minimise his welfare loss over the distribution of states in which High is reported.

The receiver’s optimal actions as a function of each type’s indifference points is obtained by finding the FOCs for (4) and (5) respectively:

\[
X_L = \frac{p}{2} T_1 + \frac{1-p}{2} T_2,
\]  
(6)

\[
X_H = \frac{1}{2} + \frac{p}{2} T_1 + \frac{1-p}{2} T_2 = \frac{1}{2} + X_L.
\]  
(7)

The optimal actions are obtained by the simultaneous solution of the first order conditions to the receiver’s optimization problems, equations (6) and (7), and each
type’s indifference point, equation (3). We derive the following results.

\[ X_L = \frac{1}{4} - pb_1 - (1 - p)b_2, \quad \text{and} \quad X_H = \frac{3}{4} - pb_1 - (1 - p)b_2, \]  

(8)

\[ T_1 = (1 - p)(b_1 - b_2) + \frac{1}{2} - 2b_1, \quad \text{and} \quad T_2 = (2 - p)(b_1 - b_2) + \frac{1}{2} - 2b_1. \]  

(9)

The restrictions on the equilibrium in the subcase when both types are moderate are \(0 < X_L < T_1 < T_2 < X_H < 1\). From \(0 < X_L\) and \(X_H < 1\) it follows that

\[-\frac{1}{4} < pb_1 + (1 - p)b_2 < \frac{1}{4}.\]  

(10)

From \(X_L < T_1 < T_2 < X_H\) it follows that

\[-\frac{1}{4} < b_2 < b_1 < \frac{1}{4}.\]  

(11)

It is easy to see that restrictions (10) follow from restrictions (11). The following result summarizes the above discussion.

**Result 1.** An equilibrium where both types are informative and moderate can be supported iff \(|b_i| < \frac{1}{4}\) for \(i = 1, 2\).

**Remark 1.** Benchmark Case. CS have shown that with linear quadratic preferences, when there is certainty over the sender’s bias, informative communication is only possible if the sender’s bias is less than \(\frac{1}{4}\). As way of comparison, in our model in the benchmark case when \(b_1 = b_2 = b\), from equation (8)

\[ X_L = \frac{1}{4} - b, \quad X_H = \frac{3}{4} - b, \quad T_1 = T_2 = \frac{1}{2} - 2b \]

and communication can only be supported if \(b < \frac{1}{4}\).

### 3.1.2 Subcase 2 — \(b_1\) extreme, \(b_2\) moderate

As Type 1’s bias increases above \(\frac{1}{4}\), \(T_1\) decreases below \(X_L\) and \(b_1\) becomes an extreme sender. The restrictions for this case are \(0 < T_1 < X_L < T_2 < X_H < 1\).
Figure 2: $b_1$ extreme, $b_2$ moderate

The receiver solves the following minimization problems for $Low$ (see Figure 2):

$$\min_{X_L} \frac{p}{2} [X_L^2 - (X_L - T_1)^2] + \frac{1-p}{2} [X_L^2 + (T_2 - X_L)^2]$$ \tag{12}

and $High$:

$$\min_{X_H} \frac{p}{2} [(X_H - T_1)^2 + (1 - X_H)^2] + \frac{1-p}{2} [(X_H - T_2)^2 + (1 - X_H)^2]$$ \tag{13}

The receiver’s optimal actions as a function of each type’s indifference points is obtained by finding the FOCs for (12) and (13) respectively:

$$X_L = \frac{1}{2} T_2 - \frac{p}{2(1-p)} T_1,$$ \tag{14}

$$X_H = \frac{1}{2} + \frac{p}{2} T_1 + \frac{1-p}{2} T_2.$$ \tag{15}

The optimal actions are obtained by the simultaneous solution of the first order conditions to the receiver’s optimization problems, equations (14) and (15), and each
type’s indifference point, equation (3).

\[ X_L = (1-p)(b_1-b_2)+\frac{(1-4b_1)(1-2p)}{2(2-p)}, \quad X_H = \frac{1}{2}+(1-p)(b_1-b_2)+\frac{(1-4b_1)(1-p)}{2(2-p)}, \]

\[ T_1 = (1-p)(b_1-b_2)+\frac{(1-4b_1)(1-p)}{2-p}, \quad T_2 = (2-p)(b_1-b_2)+\frac{(1-4b_1)(1-p)}{2-p}. \]  

The restrictions on the equilibrium in this subcase where type one is extreme and type 2 is moderate are

\[ 0 < T_1 < X_L < T_2 < X_H < 1. \]

\( T_1 > 0 \) is satisfied if

\[ b_1 < \frac{1}{2+p} - \frac{2-p}{2+p}b_2. \]

\( T_1 < X_L \) if

\[ b_1 > \frac{1}{4}. \]

\( T_2 < X_H \) if

\[ b_1 < \frac{1}{2p} + \frac{2-p}{p}b_2. \]

The following result provides a summary of when this communicative equilibrium is possible.

**Result 2.** When both types of sender are informative with \( b_1 \) extreme and \( b_2 \) is moderate a communicative equilibrium if

\[ -\frac{1}{4} < b_2 < \frac{1}{4} \quad \text{and} \quad \frac{1}{4} < b_1 < \min\left(\frac{1}{2+p} - b_2 \frac{2-p}{2+p}, \frac{1}{2p} + b_2 \frac{2-p}{p}\right). \]

Although Type 1’s bias is greater than \( \frac{1}{4} \), communication is still possible from both types as the receiver is maximizing expected utility over the reporting strategies of all sender types. In contrast to the original CS model, uncertainty over the sender’s bias allows sender types with larger biases to still be informative.

### 3.1.3 Subcase 3 — \( b_1 \) extreme, \( b_2 \) extreme

As the first type’s bias increases and the second type’s bias decreases, communication is still possible with \( T_1 < X_L \) and \( X_H < T_2 \). That is, if the biases of both types spread apart, communication can still be sustained when both types’ biases are greater in magnitude than \( \frac{1}{4} \). Type 1 reports *Low* in extremely low states, and Type 2 reports
High in extremely high states. The restrictions for this case are $0 < T_1 < X_L < X_H < T_2 < 1$. The receiver solves the following minimization problems (see Figure 3):

$$
\min_{X_L} \frac{p}{2} [X_L^2 - (X_L - T_1)^2] + \frac{1-p}{2} [X_L^2 + (T_2 - X_L)^2],
$$

and High:

$$
\min_{X_H} \frac{p}{2} [(X_H - T_1)^2 + (1 - X_H)^2] + \frac{1-p}{2} [(1 - X_H)^2 - (T_2 - X_H)^2].
$$

Using the same steps as in the previous subcases one can derive:

$$
X_L = 2p(1-p)(b_1 - b_2) + \frac{(1-4pb_1)(1-2p)}{2},
$$

$$
X_H = \frac{1}{2p} - 2(1-p)^2(b_1 - b_2) + \frac{(1-4pb_1)(2p-1)(1-p)}{2p},
$$

$$
T_1 = (1-p)(1+b_2(1-2p) - b_1(1+2p)),
T_2 = (1-p)(1+b_2(1-2p) - b_1(1+2p)) + b_1 - b_2.
$$

Using the same steps as in the previous subcases one can derive:

$$
\min_{X_L} \frac{p}{2} [X_L^2 - (X_L - T_1)^2] + \frac{1-p}{2} [X_L^2 + (T_2 - X_L)^2],
$$

and High:

$$
\min_{X_H} \frac{p}{2} [(X_H - T_1)^2 + (1 - X_H)^2] + \frac{1-p}{2} [(1 - X_H)^2 - (T_2 - X_H)^2].
$$

Using the same steps as in the previous subcases one can derive:

$$
X_L = 2p(1-p)(b_1 - b_2) + \frac{(1-4pb_1)(1-2p)}{2},
$$

$$
X_H = \frac{1}{2p} - 2(1-p)^2(b_1 - b_2) + \frac{(1-4pb_1)(2p-1)(1-p)}{2p},
$$

$$
T_1 = (1-p)(1+b_2(1-2p) - b_1(1+2p)),
T_2 = (1-p)(1+b_2(1-2p) - b_1(1+2p)) + b_1 - b_2.
$$

The restrictions on the equilibrium in the subcase where both types are extreme
are $0 < T_1 < X_L < X_H < T_2 < 1$. $T_1 < X_L$ if

$$b_1 > \frac{1}{2(1+p)} + \frac{1-p}{1+p}b_2.$$ 

$T_1 > 0$ is satisfied if

$$b_1 < \frac{1}{2p+1} - \frac{2p-1}{2p+1}b_2.$$ 

$T_2 < 1$ if

$$b_2 > -\frac{1}{3-2p} + \frac{2p-1}{3-2p}b_1.$$ 

$T_2 > X_H$ if

$$b_2 < -\frac{1}{2(2-p)} + \frac{p}{2-p}b_1.$$ 

The following result provides a summary of when this communicative equilibrium is possible.

**Result 3.** If both types are extreme, a communicative equilibrium exists if

$$\frac{1}{2(1+p)} + \frac{1-p}{1+p}b_2 < b_1 < \frac{1}{2p+1} - \frac{2p-1}{2p+1}b_2$$
and

$$-\frac{1}{3-2p} + \frac{2p-1}{3-2p}b_1 < b_2 < -\frac{1}{2(2-p)} + \frac{p}{2-p}b_1.$$ 

### 3.2 One type informative and one type uninformative

In the following subcases, there is one type whose bias is so large that she sends only one message in equilibrium - i.e. she is uninformative. There are two such subcases.

#### 3.2.1 Subcase 4 — $b_1$ uninformative, $b_2$ moderate

Type 1 is the uninformative type. As $b_1 > b_2$, this implies only Type 2 sends the message Low. In this subcase, Type 2 is a moderate sender (see Figure 4).

The receiver’s minimization problems are:

$$\min_{X_L} \frac{p}{2} [0] + \frac{1-p}{2} [X_L^2 + (T_2 - X_L)^2]$$

$$\min_{X_H} \frac{p}{2} [X_H^2 + (1 - X_H)^2] + \frac{1-p}{2} [(X_H - T_2)^2 + (1 - X_H)^2]$$

$$\min_{X_L} \frac{p}{2} [X_L^2 + (1 - X_L)^2] + \frac{1-p}{2} [(X_L - T_2)^2 + (1 - X_L)^2]$$

$$\min_{X_H} \frac{p}{2} [0] + \frac{1-p}{2} [X_H^2 + (T_2 - X_H)^2]$$
Figure 4: Type 1 is uninformative, Type 2 is moderate

Following the same steps as in previous cases to solve for $X_L$ and $X_H$:

$$X_L = \frac{1}{2(2 + p)} - \frac{2}{(2 + p)}b_2, \quad X_H = \frac{3}{2(2 + p)} - \frac{2(1 - p)}{(2 + p)}b_2$$

(24)

$$T_1 < 0, \quad T_2 = \frac{1 - 4b_2}{2 + p}.$$  

(25)

Notice that as she only sends one message, the uninformative type’s bias is irrelevant to the choice of $X_L$ and $X_H$. However, the probability of her type is relevant: as $p$ increases, $X_H$ converges to $\frac{1}{2}$.

In terms of restrictions on equilibrium in this subcase, type 1 is uninformative if $T_1 < 0$, which results in the following constraint:

$$b_1 > \frac{1}{2 + p} - \frac{(2 - p)b_2}{2 + p}.$$  

Type 2 is moderate if $X_L < T_2 < X_H$. $T_2 < X_H$ places the lower bound on $b_2$:

$$b_2 > -\frac{1}{4(1 + p)}.$$
$T_2 > X_L$ places the upper bound on $b_2$:

$$b_2 < \frac{1}{4}.$$  

This analysis is summarized by the following result.

**Result 4.** If Type 1 is uninformative and Type 2 is informative and moderate, a communicative equilibrium is supportable if $b_1 > \frac{1}{2+p} - \frac{(2-p)b_2}{2+p}$ and $-\frac{1}{4(1+p)} < b_2 < \frac{1}{4}$.

### 3.2.2 Subcase 5 — $b_1$ uninformative, $b_2$ extreme

Now consider the case when Type 1 is uninformative and Type 2 is an extreme sender, as shown in Figure 5.

![Figure 5: Type 1 is uninformative, Type 2 is extreme](image)

The receiver’s minimization problems are:

$$\min_{X_L} \frac{p}{2} [0] + \frac{1-p}{2} \left[ X_L^2 + (T_2 - X_L)^2 \right];$$  \hspace{1cm} (26)

and

$$\min_{X_H} \frac{p}{2} \left[ X_H^2 + (1 - X_H)^2 \right] + \frac{1-p}{2} \left[ (1 - X_H)^2 - (T_2 - X_H)^2 \right].$$  \hspace{1cm} (27)
In this subcase, the receiver’s optimal equilibrium actions are:

\[
X_L = \frac{1 - 4pb_2}{2(2p + 1)}, \quad X_H = \frac{1 - p}{2p} \frac{1 - 4pb_2}{2(2p + 1)}
\]  

(28)

where

\[
T_1 < 0, \quad T_2 = \frac{1 - 4pb_2}{2p + 1}.
\]

(29)

In terms of restrictions on the equilibrium, Type 1 is uninformative if \(T_1 < 0\), which results in the following constraint:

\[
b_1 > \frac{1}{2p + 1} - \frac{2p - 1}{2p + 1}b_2.
\]

Type 2 is an extreme sender if \(X_H < T_2 < 1\). \(T_2 > X_H\) is satisfied if:

\[
b_2 < -\frac{1}{4(1 + p)}.
\]

Type two is informative if \(T_2 < 1\):

\[
b_2 > -\frac{1}{2}.
\]

This analysis is summarized by the following result.

**Result 5.** If Type 1 is uninformative and Type 2 is informative and extreme, a communicative equilibrium exists if \(b_1 > \frac{1}{2p+1} - \frac{2p-1}{2p+1}b_2\) and \(-\frac{1}{2} > b_2 > -\frac{1}{4(1+p)}\).

### 3.3 No Communication

If the biases of both types are too large the only equilibrium is no communication. As the distribution of the state is uniform, the receiver’s utility maximizing action is \(\frac{1}{2}\), regardless of the message sent. This occurs in three distinct subcases.
3.3.1 Subcase 6

Type 1 sends the message *High*, and Type 2 the message *Low*. This implies $T_1 < 0$ and $T_2 > 1$, and $X_L = X_H = \frac{1}{2}$. This is the unique equilibrium iff $b_1 \geq \frac{1}{2}$ and $b_2 \leq -\frac{1}{2}$.

3.3.2 Subcase 7

Both types send the message *High*. $T_1, T_2 < 0$ and $X_H = \frac{1}{2}$. $X_L$ is unrestricted. This is the unique equilibrium iff $b_2 > \frac{1}{4}$.

All equilibria of the model with two sender types have been dealt with. In the next section, a summary of the set of equilibria under uncertainty is provided.

3.4 Characterization of the Message Space

Using the three previous subsections, Figure 6 presents a complete graphical characterization of the message space for $b_1 > b_2$ and $p = 0.5$.\(^5\) While the exact characterization depends on the probability of each type, the layout remains qualitatively unchanged for different values of $p$.

The red region corresponds to a communicative equilibrium where both types are informative; the aqua and green areas correspond to the cases where Type 1 and Type 2 are informative, respectively. Communication is not possible if $b_1$ and $b_2$ are in the yellow regions.

From section 3.3, the values $b_1$ and $b_2$ under which communication is impossible never change. On the other hand, as discussed in section 3.1, the critical level of bias at which either Type 1 or 2 becomes uninformative depends on the probability of her type, and the bias of the other type.

The following result summarizes the effect that uncertainty over the sender’s bias has on informative communication.

\(^5\)The characterization for $b_1 < b_2$ is merely the mirror image of the Figure 6 around the line $b_1 = b_2$. 

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Result 6. Under uncertainty, there is no longer a unique upper bound on the bias that can support informative communication. In contrast to Crawford and Sobel (1982), it is now possible for informative signalling if a sender’s bias is $|b_i| > \frac{1}{4}$.

The borderlines between no communication and uninformative cases provide the upper bound for the set of biases that support informative signalling from one sender type, and the borders between informative and uninformative cases provides the upper bound for the set of biases that support informative communication from all sender types.

An Example

The previous subsections have provided a complete characterization of communication when there is uncertainty about the sender’s bias. An example is now provided to more clearly show that communication can be supported for a wider range of biases under uncertainty.
Example 1. Suppose \( b_1 = 0.4, b_2 = -0.1 \) and \( p = 0.5 \). The communicative equilibrium corresponds to subcase 3.1.2. From the Appendix, the receiver’s equilibrium actions are:

\[
X_L = 0.25 \quad \text{and} \quad X_H = 0.65.
\]

Substituting the receiver’s actions into equation (3), Type 1 reports Low in states \( s \in (0, 0.05) \) and High for states \( s \in (0.05, 1) \). Type 2 reports Low for \( s \in (0, 0.55) \) and High over \( s \in (0.55, 1) \).

4 Communication versus delegation

While the focus of this section is to compare communication and delegation, does the receiver always prefers communication to no communication?

Proposition 1. The receiver’s expected utility under communication is always greater than without communication.

Proof. The proof is a revealed-preference argument. If the receiver believes that the signal is uninformative with probability 1, then his utility maximizing actions are \( X_L = X_H = \frac{1}{2} \). This set of actions will always yield the receiver an expected utility of \(-\frac{1}{4}\).

Under informative communication, the receiver’s utility maximizing actions are \( X_L < \frac{1}{2} \) and \( X_H > \frac{1}{2} \). As the receiver is still able to choose the set \( X_L = X_H = \frac{1}{2} \) and obtain on average a utility of \(-\frac{1}{4}\), not doing so must yield a strictly higher expected utility.

Turning to the receiver’s choice of communication or delegation, for each communicative subcase of section 3 the receiver’s expected utility, \( U^C \), will be computed and compared to the utility under delegation. Note that given the assumptions over the distribution of biases and the sender’s action under delegation, the receiver’s expected utility under delegation, \( U^D \), is

\[
-p|b_1| - (1 - p)|b_2|.
\]
4.1 Subcase 1 — $b_1$ and $b_2$ moderate

When both types of senders are moderate, $U^C$ is equal to negative sum of functions (4) and (5):

\[
U^C = -\frac{p}{2} \left[ X_L^2 + (T_1 - X_L)^2 + (X_H - T_1)^2 + (1 - X_H)^2 \right] - \frac{1-p}{2} \left[ X_L^2 + (T_2 - X_L)^2 + (X_H - T_2)^2 + (1 - X_H)^2 \right].
\]  (31)

Substitute values of $X_L$, $X_H$ from (8) and values of $T_1$ and $T_2$ from (9) to derive

\[
U^C = -\frac{1}{8} - pb_1^2 - (1-p)b_2^2 - (pb_1 + (1-p)b_2)^2.
\]  (32)

Communication is preferred by the receiver if $U^C > U^D$. In this subcase, communication is optimal when:

\[
\frac{1}{8} < p|b_1| + (1-p)|b_2| - pb_1^2 - (1-p)b_2^2 - (pb_1 + (1-p)b_2)^2.
\]  (33)

Analysing the above inequality, if $b_1$ and $b_2$ are biased in the same direction, then delegation always dominates communication. To see that, note that for $b_1 \neq b_2$ the following inequality holds $pb_1^2 + (1-p)b_2^2 > (pb_1 + (1-p)b_2)^2$. Labeling $\bar{b} = pb_1 + (1-p)b_2$, the RHS of inequality (33) is less than $|\bar{b}| - 2\bar{b}^2$. On the other hand, $|\bar{b}| - 2\bar{b}^2 \leq \frac{1}{8}$.

If $b_1$ and $b_2$ are biased in opposite directions, the same logics does not quite work and it is possible that communications dominates delegation. For example, for $p = 0.5$, $b_1 = 0.2$, $b_2 = -0.2$ inequality (33) is satisfied. This information is summarised in the following result.

**Result 7.** When both types have biases in the same direction, delegation dominates communication. When both types have biases in the opposite directions, communication may dominate delegation.

Sender types with opposing biases allow the receiver to take a higher Low action relative to communication only with Type 1, and a lower High action relative to communication only with Type 2. As both types’ biases are relatively small, this
shifts both types’ indifference points towards \( \frac{1}{2} \), which reduces welfare loss under communication. If \( b_1 > 0 \) and \( b_2 < 0 \) then this effect can be strong enough for communication to be optimal.

**Remark 2. Benchmark Case.** Dessein (2002) have shown that when there is certainty over the sender’s bias, delegation dominates communication whenever informative communication is possible. If \( b_1 = b_2 \) then both types have biases in the same direction, which means the above result applies, i.e. delegation dominates communication.

### 4.2 Subcase 2 — \( b_1 \) extreme, \( b_2 \) moderate

In this case \( U^C \) is equal to minus sum of functions (12) and (13):

\[
U^C = -\frac{b}{2} \left[ X_L^2 - (X_L - T_1)^2 + (X_H - T_1)^2 + (1 - X_H)^2 \right] \\
- \frac{1-p}{2} \left[ X_H^2 + (T_2 - X_L)^2 + (X_H - T_2)^2 + (1 - X_H)^2 \right].
\]

Substitute values of \( X_L, X_H \) from (16) and values of \( T_1 \) and \( T_2 \) from (17) to derive

\[
U^C = -\frac{1}{4} + \frac{p(1-p)(1-4b_1)}{(2-p)^2} + (1-p) \left( pb_1 + (2-p)b_2 + \frac{1}{2} \right) \left( b_1 - b_2 + \frac{(1-4b_1)(2-3p)}{2(2-p)^2} \right).
\]

Communication is preferred to delegation if:

\[
\frac{1}{4} - p|b_1| - (1-p)|b_2| < \frac{p(1-p)(1-4b_1)}{(2-p)^2} + (1-p) \left( pb_1 + (2-p)b_2 + \frac{1}{2} \right) \left( b_1 - b_2 + \frac{(1-4b_1)(2-3p)}{2(2-p)^2} \right).
\]

**Result 8.** For any \( b_1 \) and \( b_2 \) where Type 1 is extreme and Type 2 moderate, there exists a \( p^* \) where communication is optimal for all \( p^* < p < 1 \).

**Proof.** When \( p \) is increased, area 2 could either change or stay the same. If area stays the same then we use the following proof. Type 1 is extreme only when \( b_1 > \frac{1}{4} \), implying that for sufficiently high \( p \) the payoff from delegation is lower than \(-\frac{1}{4}\). However, from Proposition 1, the receiver’s payoff from communication must be greater than \(-\frac{1}{4}\). Consequently, given that inequality (36) is continuous, if \( p \) is sufficiently high then communication is preferred to delegation.
If the area changes, it will change to either area 5 or area 6. Specifically, both boundaries \( b_1 = \frac{1}{2p} - b_2 \frac{2-p}{2+p} \) and \( b_1 = \frac{1}{2p} - b_2 \frac{2-p}{p} \) become weaker when \( p \) increases. The first boundary gives \( b_1' = \frac{4b_2-1}{(2+p)^2} < 0 \), the second boundary gives \( b_1' = \frac{-1-4b_2}{2p^2} < 0 \).

In area 6 communication dominates delegation, see result (4.5). In area 5, a similar result holds, see result (4.4). This observation ends the proof. □

4.3 Subcase 3 — Both types extreme

For all \( b_1 \) and \( b_2 \) where both types are extreme, \( p|b_1|+(1-p)|b_2| > \frac{1}{4} \). From Proposition 1, communication is always optimal in this case.

Result 9. Communication is always optimal whenever both types are extreme.

4.4 Subcase 4 — \( b_1 \) uninformative, \( b_2 \) moderate

In this case \( U^C \) is equal to minus sum of functions (22) and (23):

\[
U^C = -\frac{p}{2} [X_H^2 + (1 - X_H)^2] - \frac{1-p}{2} [X_L^2 + (T_2 - X_L)^2 + (X_H - T_2)^2 + (1 - X_H)^2].
\]

(37)

Substitute values of \( X_L \), \( X_H \) from (24) and value of \( T_2 \) from (25) to derive

\[
U^C = \frac{2p + 1}{4(2 + p)} - \frac{4(1-p)b_2^2}{2 + p}. \tag{38}
\]

The receiver’s expected utility from communication is greater than her expected utility from delegation if:

\[
-\frac{2p + 1}{4(2 + p)} - \frac{4(1-p)b_2^2}{2 + p} > -p|b_1| - (1-p)|b_2|
\]

Result 10. For any \( b_1 \) and \( b_2 \) where Type 1 is uninformative and Type 2 moderate, there exists a \( p^* \) where communication is optimal for all \( p^* < p < 1 \).

Proof. Similarly to Subcase 4.2, Type 1 is uninformative if \( b_1 > \frac{1}{4} \), implying that for sufficiently high \( p \) the payoff from delegation is lower than \( -\frac{1}{4} \). □
4.5 Subcase 5 — $b_1$ uninformative, $b_2$ extreme

For all $b_1$ and $b_2$ where $b_1$ is uninformative and $b_2$ is extreme, $p|b_1| + (1 - p)|b_2| > \frac{1}{4}$.

From Proposition 1, communication is always optimal in this case.

**Result 11.** Communication is always optimal whenever one type is uninformative and the other extreme.

4.6 Summary

The previous subsections show that communication can be optimal in every communicative subcase. Moreover, when there is uncertainty about the sender’s bias, communication can be optimal even when both types are informative, and when both types biases are less than $\frac{1}{4}$. Figure 7 presents values of $b_1$ and $b_2$ under which communication dominates delegation, for $p = 0.5$.

![Figure 7: Regions where communication dominates delegation](image_url)

The shaded regions represent the regions where the receiver’s expected utility is greater under communication. Comparing Figure 7 to Figure 6, it is clear that
communication is optimal for a significant proportion of the values of $b_1$ and $b_2$ that support communication.

The following section provides an in-depth discussion of the results of the paper.

5 Discussion

This paper has introduced uncertainty about the sender’s bias to the Crawford and Sobel (1982) model. There are two key findings.

The first result is that communication is possible with a larger range of biases. Even when there is a single sender who may be of multiple types, there is no longer a unique upper bound on the level of bias required for communicative signalling. Indeed, sender types can be informative when their bias is greater than $\frac{1}{4}$. The second result is that the receiver’s welfare under communication can now be greater than under delegation.

The reason for these results is how uncertainty about the sender’s bias affects the incentives for strategic behaviour.

First, uncertainty over the bias can mute the strategic effects of communication. As the receiver chooses actions based on the reporting strategies of all types, sender types with opposing biases allow the receiver to take more moderate actions. In effect, the opposing biases allow the receiver to take a higher Low action and a lower High action. In response, this shifts both types’ indifference points towards $\frac{1}{2}$, which expands the set of biases that supports communication and secondly improves welfare.

The second effect of communication under uncertainty is that it allows for informative signalling from informative types, while minimising the receiver’s welfare loss from uninformative types. By retaining control over the action, an acutely biased type is prevented from taking actions that are not in the receiver’s interest. At the same time, the receiver can still maintain incentives for relatively unbiased sender types to be informative.

When the proportion of uninformative types is high, the receiver’s loss of utility from communication will be small compared to his loss of utility from relinquishing
decision rights. Communication can be optimal when the receiver believes that a sender’s bias might be high.

References


