Implications of Permanent Components for Model Design

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Many macroeconomic series seem to be best regarded as stochastically non-stationary

They have unit roots or are *integrated*

Hence have permanent components

Often have common permanent components - cointegrated

Two types of models in macro

1. Summative (reduced form) - summarize the data
2. Interpretative (structural) - interpret the data
   (a) Data-inspired structural models
   (b) Theory-inspired structural models

Summative model for series with permanent components is VECM (simplest form)

$$\Delta z_t = \alpha \beta' z_{t-1} + e_t$$

$$z_t \sim n \times 1$$

β - cointegrating vector, α loadings

$\psi_t = \beta' z_t$ is error correction (EC) term

3 Questions Examined in Seminar

1. What are implications of permanent components for *design* of data-inspired structural models
2. How do we handle permanent components in theory-inspired structural models?
3. What do permanent components in theory-inspired structural models imply about summative models?

Implications for Structural Model Design

Data-inspired interpretative model has SVECM form

$$B_0 \Delta z_t = \alpha^* \beta' z_{t-1} + \varepsilon_t$$

$$\alpha^* = B_0 \alpha, \varepsilon_t = B_0 e_t$$

Structural shocks ε_t are assumed uncorrelated

Note structural equations expected to have EC terms based on VECM

To recover shocks need to estimate coefficients in B_0. To do need instruments for Δz_t

Structural Equations: RBC Model

C =consum, K =cap stock

R = rate ret cap, Y_t =output, A_t = lev techology, hours =1

$$\ln C_t = \ln \beta + \ln C_{t+1} - \ln R_{t+1}$$

$$C_t + K_t = Y_t + (1 - \delta)K_{t-1}$$

$$R_t = \alpha \frac{Y_t}{K_{t-1}} + 1 - \delta$$

$$\ln Y_t = \alpha \ln K_{t-1} + (1 - \alpha) \ln A_t$$

$$Y_t = K_{t-1}^{\alpha} A_t{}^{1-\alpha}$$

$$Y_t = A_t^{\alpha} K_t^{1-\alpha}$$

$$y_t = a_t^* + (1-\alpha)k_t$$

$$a_t^* = (1-\alpha)a_t$$

$y_t = \ln Y_t, k_t = \ln K_t, a_t = \ln A_t$ are $I(1)$

Technology is also I(1): $\Delta a_t^* = \varepsilon_{at}$

$$\therefore \ \Delta(y_t - k_t) = \alpha(\Delta k_t - \Delta l_t) + \varepsilon_{at}$$

Hence structural equation is in differences

ε_{at} is a permanent shock

Note : No EC term in equation

EC term $y_{t-1} - k_{t-1}$ can be used as an instrument for $(\Delta k_t - \Delta l_t)$

Simulate data from RBC Model in Ireland

$cor[(\Delta k_t - \Delta l_t), (y_{t-1} - k_{t-1})] \quad = .73$

Excellent instrument

In contrast $(\Delta k_{t-1} - \Delta l_{t-1})$ weakly correlated with $(\Delta k_t - \Delta l_t)$

Result applies more generally

Any structural equation designated to have a permanent shock (ε_{at}) *will have variables in differences and no ECM term*

Permanent components therefore have implications for data-inspired structural model design and shock identification

Pagan and Pesaran establish result above and use to synthetize many studies in literature e.g. Blanchard-Quah supply/demand identification

Important point: If VECM $\Rightarrow r$ co-integrating vectors

$\Rightarrow n-r$ common permanent components

$\Rightarrow n-r$ permanent shocks (e_t)

$\Rightarrow n-r$ *permanent structural* shocks (ε_t)

So need to specify which $n-r$ structural equations these appear in

Then those equations must be estimated in differences and lagged EC terms are instruments

$\therefore$ Need to think through how one designs the structural model to ensure this

Need to think structurally not in terms of VAR/VECM

Permanent Components and the Design of Theory Inspired Models

Focus on Lubik and Shorfheide (JME 2007)

Well known DSGE model of small open economy

Feature was allowed technology shocks to be integrated processes

Theory-inspired model: Based on Gali-Monicelli

Consumption det by max separable function in consumption C_t and hours H_t (all per capita)

$$\max E_t \sum_{j=0}^{\infty} \beta^j (h(C_{t+j}) - g(H_{t+j}))$$

s.t.

$$P_t C_t + B_{t+1} = W_t H_t + R_{t-1} B_t$$

(R_t rate of return, $B_t = p.c.$ bond, $W_t =$ p.c. hourly wage, H_t is p.c. hours)

Two common choices for $h(.)$

(i) Log utility $\ln C_t$

(ii) Power utility $\frac{C_t^{1-\sigma}}{1-\sigma}$

What do we do when C_t has a permanent component?

Log Utility case

Normalize C_t by some exogenous $I(1)$ process Z_t such that $\phi_t = \ln C_t - \ln Z_t$ is $I(0)$ $i.e. \Phi_t = \frac{C_t}{Z_t}$

Then max w.r.t. ϕ_t

Objective function in this case is then

$$\max E_t \sum_{j=0}^{\infty} \beta^j (\phi_{t+j} + \ln Z_{t+j} - g(N_{t+j}))$$

$$\max E_t \sum_{j=0}^{\infty} \beta^j (\phi_{t+j} + \ln Z_{t+j} - g(N_{t+j}))$$

Could just max w.r.t.

$$E_t \sum_{j=0}^{\infty} \beta^j (\phi_{t+j} - g(N_{t+j}))$$

since $E_t \sum_{j=0}^{\infty} \beta^j \ln Z_{t+j}$ bounded and doesn't depend on ϕ_{t+j}

Solution for C_t invariant to Z_t :

$$C_t^{-1} = \beta E_t C_{t+1}^{-1} \frac{P_t}{P_{t+1}} R_t.$$

Often choose Z_t as log of technology

But could choose many other items- income, $e^{c_t^P}$ etc

Power utility Case

Follow re-scaling strategy

$$E_t \sum_{j=0}^{\infty} \beta^j \left[\frac{(\frac{C_t}{Z_t})^{1-\sigma} Z_t^{1-\sigma}}{1-\sigma} - g(N_t) \right],$$

Euler equation

$$C_t^{-\sigma} = \beta E_t C_{t+1}^{-\sigma} \frac{P_t}{P_{t+1}} R_t.$$

Doesn't depend upon Z_t

Problem is that don't get balanced growth solution

i.e. $c_t = \ln C_t, z_t = \ln Z_t$ don't cointegrate

z_t cointegrates with output in LS model

Change utility function to del Negro/ Schforheide (basis of LS model)

$$E_t \sum_{j=0}^{\infty} \beta^j \left[\frac{(\frac{C_t}{Z_t})^{1-\sigma}}{1-\sigma} - N_t \right]$$

Euler equation is

$$C_t^{-\sigma} = -\beta E_t C_{t+1}^{-\sigma} \frac{P_t}{P_{t+1}} (\frac{Z_t}{Z_{t+1}})^{1-\sigma} R_t,$$

Does give balanced growth

BUT

Answer for consumption depends on what we choose to normalize with

Exception is $\sigma = 1$ (log utility)

Seems unsatisfactory

Why this dependence?

Ans is that each Z_t is effectively a different theory about consumer choice since utility depends on C_t/Z_t

Thus if Z_t = permanent level of consumption (which is very common)

Says consumers are only concerned about the *transitory* level of consumption not the level of consumption per se

Thus what Z_t is appropriate is something that you need to test with data

Re-scaling has real effects

Suggests that if you worry about this re-scaling issue you need to use log utiliy

In fact many do

But most stationary models use power utility

Extensions to I(1) case (such as Lubik-Schorfheide) have fallen into the trap

What do permanent components in theory-inspired structural models imply about summative models?

In one-factor DSGE models e.g RBC model, all I(1) variables (consumption, investment...) have same permanent component

This is permanent component of technology

So if scale by this variables have been converted to transitory components ("gaps")

System can then be expressed in z_t^T and Δa_t

Solves to give a VAR in z_t^T and Δa_t(a_t is latent)

This is a **latent variable VAR**

VAR Model in z_t^T, Δa_t will be called the TF (Transitory form)

The TF is unlikely to give a VAR in z_t^T alone

If z_{1t}, z_{2t} are variables in VAR

Eliminating z_{2t} only gives a VAR in z_{2t} if (Fukac/Pagan)

(a) z_{2t} is related to z_{1t} via an identity

(b) z_{2t} has no dynamics

In theory-inspired models it is impossible to eliminate Δa_t from VAR and result in a VAR in z_t^T

Will need to be a VARMA in z_t^T

VARMA can be estimated by setting up a state space form

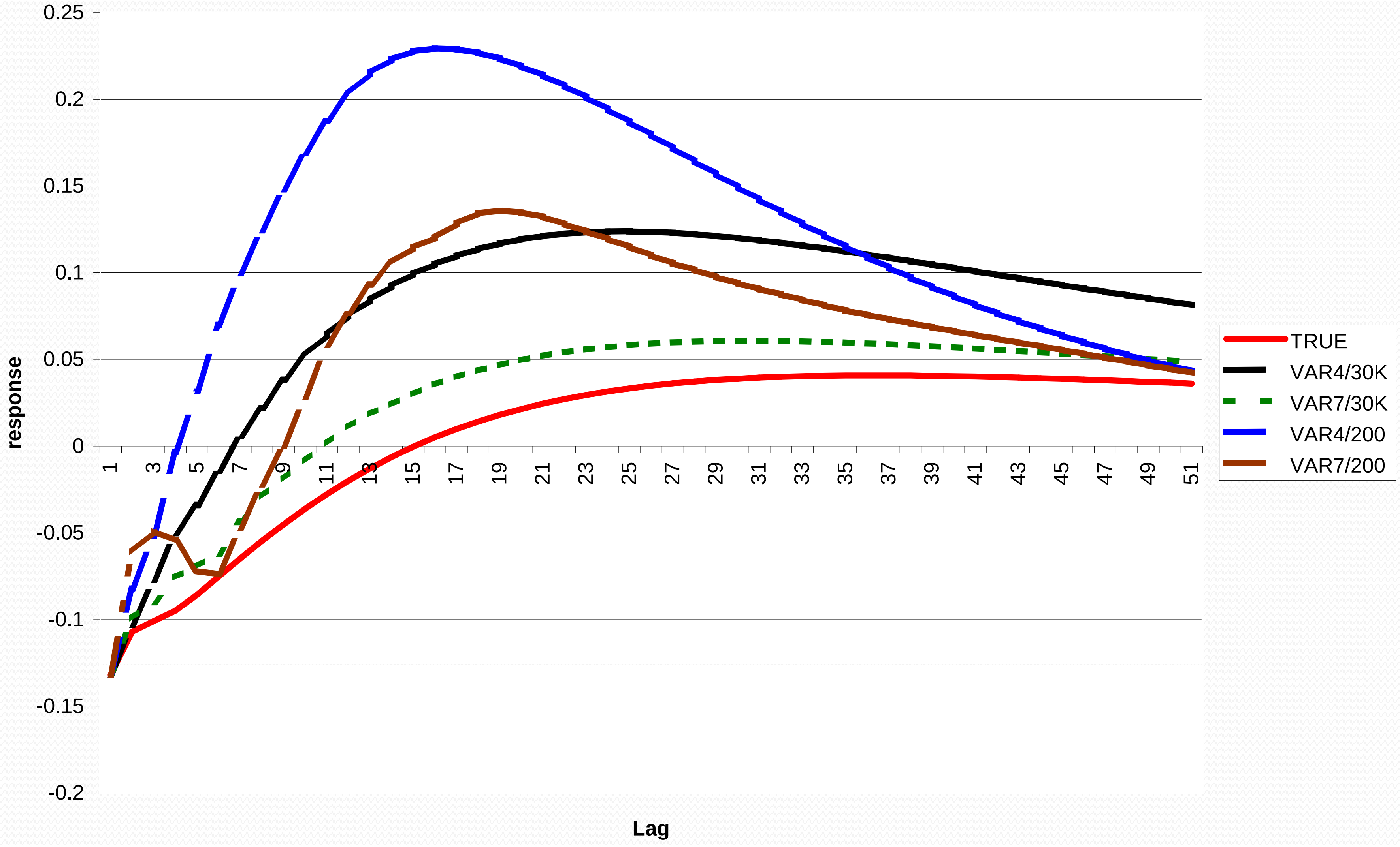
(i) VAR Dynamics for $z_t^T, \Delta a_t$

(ii) $\Delta z_t = \Delta z_t^T + \Delta a_t$ as obs eqn

Estimate in DYNARE

Might approximate with VAR but often incredibly high order for realistic models (VAR(50) for mini-BEQM)

Figure 3 Impulse Responses of Real Exchange Rate to Demand Shocks



What is relation between VAR in $z_t^T, \Delta a_t$ and a VECM?

$$z_t^+ = \begin{bmatrix} z_t \\ \Delta a_t \end{bmatrix}$$

$$VECM : \Delta z_t^+ = \alpha \psi_{t-1}^+ + e_t,$$

$$SVECM : \bar{B}_0 \Delta z_t^+ = \bar{B}_0 \alpha \psi_{t-1}^+ + \varepsilon_t$$

$$\begin{aligned} \psi_t^+ &= \beta' (z_t^{+P} + z_t^{+T}) \\ &= \beta' z_t^{+T} \\ &= \beta_1' z_t^T + \beta_2 a_t^T, \end{aligned}$$

Assume

$$\Delta a_t = \rho \Delta a_{t-1} + \varepsilon_{at}$$

$$\Rightarrow a_t^T = \phi \Delta a_t$$

$$\begin{bmatrix} \psi_t^+ \\ \Delta a_t \end{bmatrix} = \begin{bmatrix} \beta_1' z_t^T + \beta_2 \phi \Delta a_t \\ \Delta a_t \end{bmatrix}$$

$$= \begin{bmatrix} \beta_1' & \beta_2 \phi \\ 0 & 1 \end{bmatrix} \begin{bmatrix} z_t^T \\ \Delta a_t \end{bmatrix}$$

$$= Q \begin{bmatrix} z_t^T \\ \Delta a_t \end{bmatrix}$$

Standard case: $\beta_1 = I_n$ (RBC model - cointegrating vectors all (1 -1))

$$\Rightarrow \begin{bmatrix} \beta_1' & \beta_2'\phi \\ 0 & 1 \end{bmatrix} \text{ is non-singular}$$

Also have

$$\begin{bmatrix} \psi_t^+ \\ a_t \end{bmatrix} = \begin{bmatrix} \beta_1' & \beta_2 \\ 0 & I \end{bmatrix} \begin{bmatrix} z_t \\ a_t \end{bmatrix} = Hz_t^+$$

H is non-singular

Hence SVECM written as

$$\bar{B}_0 H^{-1} H \Delta z_t^+ = \bar{B}_0 H^{-1} \begin{bmatrix} \Delta\psi_t^+ \\ \Delta a_t \end{bmatrix}$$

$$= \bar{B}_0 \alpha \psi_{t-1}^+ + \varepsilon_t.$$

$$\bar{B}_0 H^{-1} \begin{bmatrix} \Delta\psi_t^+ \\ \Delta a_t \end{bmatrix} = \bar{B}_0 \alpha(\beta_1' z_{t-1}^T + \beta_2 \phi \Delta a_{t-1}) + \varepsilon_t$$

$$B_0 \begin{bmatrix} z_t^T \\ \Delta a_t \end{bmatrix} = D_1 z_{t-1}^T + D_2 \Delta a_{t-1} + \varepsilon_t \ (TF)$$

Implications

$$SVECM : \bar{B}_0 \Delta z_t^+ = \bar{B}_0 \alpha \psi_{t-1} + \varepsilon_t$$

$$TF : B_0 \begin{bmatrix} y_t^T \\ \Delta a_t \end{bmatrix} = D_1 z_{t-1}^T + D_2 \Delta a_{t-1} + \varepsilon_t$$

$B_0 = \bar{B}_0 H^{-1} Q$

$H^{-1}Q$ is triangular (H and Q have property in the standard case)

$\therefore$triangular $\bar{B}_0 \Rightarrow B_0$ is triangular

shocks in TF may be different because of Δa_{t-1}

Special case: Δa_t is white $\Rightarrow \phi = 0 \Rightarrow D_2 = 0$

Then shocks in TF and SVECM are the same

Basically estimating a TF involves estimating z_t^T by Beveridge-Nelson

Can't use Hodrick-Prescott

That would cause specification error in equations (Fukac/Pagan)

Equivalence means can move between VECM and model put in transitory form

Useful for many purposes

Sign restrictions from DSGE model can be imposed on VECM residuals

New Keynesian model has output gap

Often measured as deviation of log output from permanent (supply) component

So transitory output = gap

Output gap appears in New Keynesian models

$\therefore$ Basically NK model is a TF

$\therefore$ Result above says we can re-write NK model as a VECM in log output *and* permanent output component

Latter is a latent variable so need to estimate a VECM with latent variable via state space form

Conclusions

- Permanent components have implications for the form of structural equations. Many examples where this has been invalidly forgotten.
- Need some care in defining utility function. Best solution is to normalize with permanent level of consumption since this equals the permanent level of technology and so consumers can only change the extent to which consumption departs from this "bliss point"
- VECM methods used by macroeconometrics; gap models used by applied macro theorists. Connection has not been examined but we have shown that they are isomorphic