

# Free entry and social efficiency in an open economy

Arghya Ghosh, Jonathan Lim, and Hodaka Morita\*

## Abstract

Is free entry desirable for social efficiency? We examine this question in an open economy context under a Cournot setup. Specifically, we consider a ‘home market’ where both home and foreign firms compete simultaneously. While free entry always leads to “excessive entry” in a closed economy setup (Mankiw and Whinston (1987), Suzumura and Kiyono (1986)), we find that free entry can be socially insufficient once there are some foreign firms. For linear demand, we find that as trade is liberalized, “insufficient entry” becomes more likely. This implies that entry regulation may not be as desirable as economies become more open.

---

\* School of Economics, University of New South Wales, Sydney, NSW 2052, AUSTRALIA.

e-mail: Ghosh: a.ghosh@unsw.edu.au, Lim: jonathanlim@y7mail.com, Morita: h.morita@unsw.edu.au.

# 1 Introduction

Is free entry desirable for social efficiency? Since the answer has important implications for entry regulation policies, not surprisingly, this question has been extensively analyzed in industrial organization literature. However, such analysis have almost exclusively been conducted in a closed economy setting. Thanks to the reduction in trade barriers across the world, countries are now more open than ever before. Reexamining this question in a open economy set up our analysis provides a new perspective on the desirability of free entry. We consider a homogenous product Cournot oligopoly where both domestic and foreign firms compete in the home market. We show that in the presence of foreign firms, free entry of home firms can be insufficient. In other words, the number of home firms that enter in a free-entry equilibrium can fall short of the number that maximizes home welfare.

This insufficient entry result is in contrast to previous findings obtained from a closed economy set up. In a Cournot oligopoly with homogenous products, Mankiw and Whinston (1986) and Suzumura and Kiyono (1987) have shown that if output per firm falls as the number of firms in the industry increases (a “business stealing effect”), the level of entry in a free-entry equilibrium is always socially excessive (see also von Weizsacker, 1980; Perry, 1984). This result holds in our framework as a special case where there are no foreign firms. However, except for this special case, we find that there always exists parameterizations such that entry improves welfare.

Consider an oligopolistic industry with home and foreign firms. To model foreign presence in a simple fashion, we assume that foreign firms exercise voluntary export restraint and produce a fixed amount of output. Furthermore, there exists a large number of potential home firms, which can enter by paying a fixed cost. Post-entry competition is Cournot. The entry of a domestic firm produces two effects. When a domestic firm enters, total output increases, but existing firms cut back their production. Thus, a part of the business of the entrant is simply stolen from other domestic firms. The stolen business is valued by the entrant since it enhances profits. However, the value of this stolen business is simply a transfer of surplus from existing firms to the entrant, thus does not contribute to total surplus. Hence, the ‘business-stealing effect’ works in the direction of excessive private incentive for entry.

In our framework, there is an additional effect. The new domestic entry also induces a reduction in price, which leads to a transfer in surplus from producers to consumers. Transfer

from domestic producers to domestic consumers does not matter for total domestic surplus. However, due to the presence of foreign firms, part of the surplus transferred is from foreign producers to domestic consumers. This leads to an increase in total domestic surplus. Since the marginal entrant cannot capture this as its own profit, it ignores this positive consequence on total domestic surplus. This effect, which does not arise in the closed economy setting, works in the direction of insufficient private entry. We find that if this new effect dominates the business stealing effect, then the number of home firms in the free entry equilibrium can be socially insufficient rather than excessive. Anticompetitive industrial policy, such as entry regulation, have been employed in several countries (e.g., Japan, South Korea, India, China). It has often been argued that the “excess-entry theorem”, i.e., the finding that free entry leads to socially excessive number of firms, can provide a justification for such policies. Our contribution is to demonstrate that previous justification may not necessarily be valid under this new trading environment.

In sections 2 and 3 we focus on VER (as the trade instrument) to illustrate our insufficient entry result in simplest possible way. Note that foreign firms do not act strategically. In section 4 we extend our analysis to a setting where foreign firms also choose quantities strategically, and faces a tariff of  $t$  per unit of output instead of VER. We find that if tariff rate is lower than a threshold then free entry leads to a insufficient number of domestic firms. This finding suggest that as the economies become more open entry regulation of domestic firms might no longer improve welfare. Section 5 extends this model by allowing for the free entry of foreign firms. Under this framework we also find consistent results to the previous analysis. That is, free entry entry of domestic firms may lead to socially insufficient entry.

Our paper is not the first to point out that the level of entry in the free-entry equilibrium might be socially insufficient. It is well known that under the presence of product diversity where consumers prefer variety, free entry can result in a socially insufficient number of firms (See Spence, 1976; Dixit and Stiglitz, 1977). Ghosh and Morita (2007) has shown that insufficient entry can occur even in homogeneous product markets, when firms’ interactions with other firms in vertically related industries are taken into account. We show that, even in the absence of product differentiation and vertical relationships, free entry can be socially insufficient provides some firms are foreign. Our theoretical analysis suggest that entry regulation of the home firm at the margin - a welfare improving policy in a closed economy - may be detrimental to welfare in an open economy.

## 2 Model

We consider an industry consisting of home and foreign firms selling a homogenous product in the home market. The firms face the inverse demand given by  $P(Q)$  where  $Q(> 0)$  denotes the aggregate industry output. We assume that (i)  $P(Q)$  is continuously differentiable as often as it is required and  $P'(Q) < 0$  for all  $Q$ , and (ii)  $P_0 > c > P_\infty$  where  $P_0 \equiv \lim_{Q \rightarrow 0} P(Q)$  and  $P_\infty \equiv \lim_{Q \rightarrow \infty} P(Q)$ . Each home firm has a marginal cost  $c \geq 0$ . In addition for the home firm there is an entry cost  $K(> 0)$ . We further assume that the aggregate output of the foreign firm is fixed at  $Y > 0$ , which captures the voluntary export restraint (VER) in our model. Voluntary export restraints (VER) are discriminatory export restraints typically ‘voluntarily’ administered by the exporting countries at the request of their importing counterparts.<sup>1</sup> Note that  $Y$  serves as a proxy for the openness of the home industry. In particular, a larger  $Y$  implies that the economy is more open.

We consider the two stage game described below. In the first stage, a large number of identical potential domestic firms exist, each of whom must decide whether or not to enter. Stage 2 involves Cournot competition in which each profit-maximising firm chooses quantity taking the rival firms output and the VER as given. We consider the second-best problem faced by the home government who can control the number of home firms that can enter, but cannot control their post-entry output choices. This is in line with standard practice in this literature (see, for example, Mankiw and Whinston (1986) and Suzumura and Kiyono (1987)). The home government’s objective is to select the number of home firms that maximises the total surplus of its own country, which is defined as gross benefits to consumers less the sum of production costs, entry costs and revenue accruing to the foreign firm.

## 3 Voluntary Export Restraint

We consider the Subgame Perfect Nash Equilibria (SPNE) of the model described in the previous section. We first analyse the model without specifying the functional form of the inverse demand function  $P(Q)$  and identify the condition under which insufficient entry

---

<sup>1</sup>Until recently VERs were used extensively by many countries including United states (Dean and Gangopadhyay, 1991; Ishikawa, 1998;), European Union (Hamilton, 1991), United Kingdom (Brenton and Winters, 1993), Taiwan and South Korea (Song, 1996).

occurs in the first stage (Proposition 1). Then using a linear inverse demand function, we demonstrate that insufficient entry occurs under a range of parameterizations (Proposition 2).

### 3.1 General Demand

Suppose  $N$  firms have entered in stage 1. Now consider stage 2, i.e. the Cournot competition amongst the  $N$  home firms. Each firm  $i (= 1, 2, 3, \dots, N)$  chooses output  $x_i (\geq 0)$  to maximize its profit  $[P(x_i + \sum_{j \neq i} x_j + Y) - c]x_i$ , taking other firms' output and VER as given. If  $x_i > 0$  for all  $i$ , the standard first order conditions are

$$P \left( Y + x_i + \sum_{j \neq i} x_j \right) - c + P' \left( Y + x_i + \sum_{j \neq i} x_j \right) x_i = 0. \quad (1)$$

We make the following assumption which is satisfied by a number of standard inverse demand functions.

**Assumption 1:** (i)  $\lim_{Q \rightarrow 0} [P(Q) + QP'(Q)] = P_0$  and (ii)  $2P'(Q) + QP''(Q) < 0$  for all  $N$  satisfying  $N \geq 1$ .

**Proof:** See Appendix.

Assumption 1 guarantees that the system of equations (1) yields a unique solution  $\hat{x}_1 = \hat{x}_2 = \dots \equiv \hat{x} (> 0)$ . This characterizes the unique (and symmetric) equilibrium of the Stage 2 subgame.

Each home firm's profit is given by  $\tilde{\pi}(N) - K \equiv \pi(N)$ , where  $\tilde{\pi}(N) = [P(Y + N\hat{x}) - c]\hat{x}$ . and  $N$  is the number of home firms that enter at stage 1. We treat the number of firms as a continuous variable. Since  $\hat{x}$  is implicitly defined by  $P(Y + N\hat{x}) - c + P'(Y + N\hat{x})\hat{x} \equiv 0$ , we have that  $\hat{x}$  is a continuously differentiable function of  $N$ .

**Lemma 1:** In the equilibrium of the Stage 2 subgame, each home firm's profit,  $\pi(N)$ , is strictly decreasing in  $N$  for all  $N > 1$ .

**Proof:** See Appendix.

To ensure that at least one firms enters in the equilibrium we assume the following.

**Assumption 2:**  $K \leq \bar{K} \equiv \tilde{\pi}(1)$ .

We define the free-entry number of home firms to be the largest  $N$  ( $\geq 1$ ) such that  $\pi(N) = 0$ . Let  $N_f$  denote the free entry number of home firms. It can be shown that a unique  $N_f$  exists.

We now consider the second-best problem faced by the home government whose objective is to maximize the surplus accruing to the home country by controlling the number of home firms that enter. The total surplus in the SPNE of the stage 2 subgame, where  $N$  home firms have entered in the previous stage, is:

$$\int_0^{Y+N\hat{x}} P(s)ds - Nc\hat{x} - NK,$$

where  $Y$ , as mentioned before, is the fixed amount of foreign output in the home industry and  $\hat{x}$ , is the quantity produced by the domestic firms in the stage 2 Cournot equilibrium.

Subtracting the consumer expenditure on foreign output,  $P(Y + N\hat{x})Y$  from the total surplus, gives,

$$\int_0^{Y+N\hat{x}} P(s)ds - Nc\hat{x} - NK - P(Y + N\hat{x})Y \equiv W(N),$$

where  $W(N)$  refers to the home country's total surplus. Note, the home government chooses  $N = N^*$  to maximise  $W(N)$ . This implies  $W'(N)|_{N=N^*} = 0$ , where

$$\begin{aligned} W'(N) &= P(Y + N\hat{x}) \left[ N \frac{\partial \hat{x}}{\partial N} + \hat{x} \right] - \left( c\hat{x} + Nc \frac{\partial \hat{x}}{\partial N} \right) - K - \left( \frac{dP(Y + N\hat{x})}{dN} \right) Y, \\ &= \pi(N) + (P(Y + N\hat{x}) - c)N \frac{\partial \hat{x}}{\partial N} - \left( \frac{dP(Y + N\hat{x})}{dN} \right) Y. \end{aligned}$$

Recall that  $\pi(N)$  is strictly decreasing in  $N$  for all  $N > 1$  (Lemma 1). Then  $N_f < N^*$  holds if and only if  $\pi(N^*) < \pi(N_f) \equiv 0$ . Given the condition  $W'(N)|_{N=N^*} = 0$ , we have the following implication:

**Proposition 1:** Suppose that Assumption 1 holds. Then the number of domestic firms in the free-entry equilibrium is insufficient if and only if the following holds:

$$(P(N\hat{x} + Y) - c)N \frac{\partial \hat{x}}{\partial N} - \left( \frac{dP(Y + N\hat{x})}{dN} \right) Y > 0 \quad (2)$$

where all the expressions are evaluated at  $N = N^*$ .

Proposition 1 states the condition under which socially insufficient entry can occur. As in Mankiw and Whinston (1986), we say that the business-stealing effect is present at  $N = \tilde{N}$  if  $\frac{\partial \hat{x}}{\partial N}|_{N=\tilde{N}} < 0$ . Since  $P(N\hat{x} + Y) - c > 0$ , the first term of the left-hand side of the inequality (2) is negative in the presence of the business stealing effect at  $N = N^*$ . If there are no exports ( $Y = 0$ ), then only the ‘business stealing effect’ is present and inequality (2) cannot hold. This confirms the findings of Mankiw and Whinston (1986) and Suzumura and Kiyono (1987) i.e. free entry leads to socially excessive number of firms. However, if  $Y > 0$ , the inequality can hold since the second term,  $-\left(\frac{dP(Y+N\hat{x})}{dN}\right)Y$ , is strictly positive. Thus, while in a closed economy setting free entry leads to socially excessive number of firms, in an open economy setting free entry number of home firms might be insufficient.

First we explain the standard “excess-entry” result obtained in a closed economy setting. Assume that  $Y = 0$ . The increment in social cost due to entry is  $K$ , while the incremental social benefit is  $(P(N\hat{x}) - c)(\hat{x} + N \frac{\partial \hat{x}}{\partial N})$ . Thus the net increment in social benefit due to entry is

$$\begin{aligned} & [(P(N\hat{x}) - c)\hat{x} - K] + (P(N\hat{x}) - c)N \frac{\partial \hat{x}}{\partial N}, \\ & = \pi(N) + (P(N\hat{x}) - c)N \frac{\partial \hat{x}}{\partial N}. \end{aligned}$$

Note that the net private benefit to the entrant is  $\pi(N)$ . The entrant ignores the negative consequence of its entry captured by the term  $(P(N\hat{x} + Y) - c)N \frac{\partial \hat{x}}{\partial N}$ . In the presence of business-stealing effect, (i.e.,  $\frac{\partial \hat{x}}{\partial N} < 0$ ), the increment in net benefit due to entry is negative at  $N = N_f$  (where  $\pi(N) = 0$ ). In other words, the number of firms entering in the free entry equilibrium is socially excessive.

Now, let us consider the open economy setting with  $Y > 0$ . In this case, the net increment in total domestic surplus is

$$\pi(N) + (P(N\hat{x} + Y) - c)N \frac{\partial \hat{x}}{\partial N} + \left( -\frac{dP(Y + N\hat{x})}{dN} Y \right).$$

As before, the entrant only cares about  $\pi(N)$  and ignores the negative effect on other home producers  $(P(N\hat{x} + Y) - c)N \frac{\partial \hat{x}}{\partial N}$ . However, it also ignores a positive effect induced by entry.

Entry lowers price which in turn increases consumer surplus. The term  $-\frac{dP(Y+N\hat{x})}{dN}Y$  captures the transfer of surplus from foreign producers to domestic consumers. Consider a closed economy consisting of only domestic firms (i.e.,  $Y = 0$ ) this transfer of surplus from producers to consumers do not add to total surplus, while in an open economy it does. If the ignored positive effect,  $-\frac{dP(Y+N\hat{x})}{dN}Y$ , is greater than the negative effect,  $P(N\hat{x} + Y) - c)N\frac{\partial\hat{x}}{\partial N}$ , then the free entry number of home firms is socially insufficient at the margin.

The logic behind Proposition 1 can also be explained by using a diagram (see Figure 1). The marginal entrant produces  $\hat{x}$  as denoted by the segment  $HJ$  implying profits as  $EGJH$  i.e.  $(P(\cdot) - c)\hat{x}$ . The business stealing effect,  $(P(\cdot) - c)N\frac{\partial\hat{x}}{\partial N} < 0$ , which does not contribute to total home surplus is captured by the area  $EFIH$ . Given the entrant ignores the business stealing effect the social contribution of the marginal entrant,  $(P(\cdot) - c)(\hat{x} + N\frac{\partial\hat{x}}{\partial N})$ , is given by the area  $FGJI$ . Observe that  $FGJI < EGJH$  which implies the business stealing effect works in the direction of excessive entry.

Entry induces a reduction in price, denoted by  $AC$ , which leads to an increase in consumer surplus given by the area  $AKGC$ . The area  $AKGC$  consists of the following components;  $KGF$ ,  $BKFD$  and  $ABDC$ . The area  $KGF$  is of second order thus we ignore this effect. The area  $BKFD$ , captures the transfer of producers surplus to consumer surplus which doesn't add to total surplus. Finally, we have  $ABDC$  (i.e.  $-\frac{\partial P(\cdot)}{\partial N}Y$ ) capturing the transfer of surplus from foreign producers to consumer, which adds to total domestic surplus. The marginal entrant ignores this positive effect of entry which works in the direction of insufficient entry. Thus for insufficient entry to occur we require  $ABDC > EFIH$  at the free entry equilibrium.



Figure 1

### 3.2 Linear Demand

Thus far we have discussed the possibility of insufficient entry. However, does insufficient entry actually occur? In this subsection we consider linear inverse demand function and demonstrate that insufficient entry occurs under a wide range of parameterizations. It is easy to check that the linear demand function of the form

$$P = a - bQ$$

satisfies Assumption 1. Analogous to Assumption 2 we assume that  $K \leq \bar{K} \equiv \frac{(a-bY-c)^2}{4b}$ , which ensures that at least one firm enters in the equilibrium. Routine calculations give

$$x_i = \hat{x} = \frac{a - bY - c}{b(N + 1)}, \quad \pi(N) = \frac{1}{b} \left[ \frac{a - bY - c}{(N + 1)} \right]^2 - K,$$

as the equilibrium output and profits respectively. Observe that  $\frac{\partial \hat{x}}{\partial N} < 0$  for all  $N \geq 1$ , thus the business stealing effect is present for all  $K < \bar{K}$ . Also, observe that  $\pi(N)$  is strictly decreasing in  $N$ . Setting  $\pi(N) = 0$  and solving for  $N$  we get the free entry number of firms:

$$N_f = \frac{a - bY - c}{\sqrt{bK}} - 1.$$

Furthermore we find that for linear demand  $W(N)$  is strictly concave. We denote the solution to  $W'(N) = 0$  as  $N^*$  and assume that  $K < \frac{(a-c)^2 - b^2Y^2}{8b} \equiv K^*$  to ensure that  $N^* > 1$ . We find that insufficient entry occurs globally if and only if

$$W'(N_f) \equiv \frac{(a - bY - c)(a + N_f bY - c)}{b(N_f + 1)^3} - K > 0.$$

Substituting  $N_f = \frac{a - bY - c}{\sqrt{bK}} - 1$  into the above expression we have that insufficient entry occurs if and only if,

$$K > \frac{(a - 2bY - c)^2}{b} \equiv \tilde{K}, \quad (3)$$

where  $\tilde{K} > 0$ . To ensure that the environment is conducive for entry we require,

$$\tilde{K} < \min\{\bar{K}, K^*\} \equiv \check{K}.$$

It is straightforward to establish that:

$$\tilde{K} < \min\{\bar{K}, K^*\} \Leftrightarrow Y > \frac{a - c}{3b}$$

which implies as long as  $Y > \frac{a-c}{3b}$ , there is a possibility of insufficient entry. Now we are ready to present the main result in this subsection.

**Proposition 2:** Suppose the inverse demand function is given by  $P = a - bQ$  where  $a > 0, b > 0$ . For all  $Y > \frac{a-c}{3b}$ , there exists  $\tilde{K}(Y) > 0$  such that for all  $K < \tilde{K}(Y)$ , at least one firm enters in the market, and *insufficient entry* occurs despite the presence of business stealing effect. Furthermore,  $\tilde{K}(Y)$  is strictly decreasing in  $Y$ .

Proposition 2 confirms that there exists parameterizations such that insufficient entry occurs globally. Note that  $Y$  needs to be greater than a threshold value for insufficient entry to occur. The finding that  $\tilde{K}(Y)$  is strictly decreasing in  $Y$  suggests that if  $Y$  is higher than threshold, an increase in foreign presence makes insufficient entry more likely. The reasoning

is as follows. For a given aggregate output level, the higher the share of  $Y$  in aggregate output the lesser is the concern about business stealing (as there is not much home business to steal). Moreover, the higher the share of  $Y$ , the greater is the transfer of surplus,  $\frac{-dP(\cdot)}{dN}Y$ , from foreign producers to domestic consumers. Both these suggest that the higher the  $Y$  the more likely is insufficient entry.

## 4 Tariff

So far we have focused on VER to illustrate the idea of insufficient entry result in a simple fashion. However, by focusing on VER we miss one of the motivation for trade protection: government revenues. Now we consider the other popular trade instrument: tariffs. Intuitively, the presence of tariff revenues (extracted from the foreign firms) reinforces the incentive for entry regulation of home firms.

We first consider a set up where domestic and foreign firms compete in quantities and identify the condition under which insufficient entry may occur. Later, we consider a linear demand function and show that insufficient entry can occur under a wide range of parameterizations. We find that as tariffs decline, the incentive for restricting entry of home firms decline.

### 4.1 General Demand

We consider Cournot competition among the  $N$  domestic firms and  $M$  foreign firms in the second stage of the game. Each domestic firm  $d(= 1, 2, 3, \dots, N)$  chooses  $x_d(> 0)$  to maximize  $[P(x_d + \sum_{j \neq d} x_j + \sum_{f=1}^M y_f) - c_d]x_d$  while each foreign firm  $f(= 1, 2, 3, \dots, M)$  chooses  $y_f(> 0)$  to maximize its  $[P(y_f + \sum_{j \neq f} y_j + \sum_{d=1}^N x_d) - c_f - t]y_f$ . The standard first order conditions are:

$$P(x_d + \sum_{i \neq d} x_i + \sum_{f=1}^M y_f) - c_d + P'(x_d + \sum_{i \neq d} x_i + \sum_{f=1}^M y_f)x_d = 0; \quad d \in \{1, 2, \dots, N\}, \quad (4)$$

$$P(y_f + \sum_{j \neq f} y_j + \sum_{d=1}^N x_d) - c_f - t + P'(y_f + \sum_{j \neq f} y_j + \sum_{d=1}^N x_d)y_f = 0; \quad f \in \{1, 2, \dots, M\}. \quad (5)$$

We make the following assumption (satisfied by a number of standard inverse demand functions) which together with Assumption 1(i) ensures existence and uniqueness of the equilibrium.

**Assumption 3:**  $(M + N + 1)P'(Q) + QP''(Q) < 0$  for all  $M, N$  satisfying  $N + M \geq 1$ .

Condition analogous to Assumption 3 has been identified in the literature as key conditions for uniqueness of the Cournot equilibrium (see for example, Vives, 1999). If  $c_d \in (0, P_0)$ , and  $(c_f + t) \in (0, P_0)$ , Assumption 3 guarantees that the system of equations (4) and (5) yields a unique solution for domestic and foreign firm output. These solutions are respectively denoted by  $\hat{x}_1 = \hat{x}_2 = \dots \equiv \hat{x} (> 0)$  and  $\hat{y}_1 = \hat{y}_2 = \dots \equiv \hat{y} (> 0)$ , for the domestic and foreign firms. Since  $\hat{x}$  and  $\hat{y}$  are implicitly defined by  $P(N\hat{x} + M\hat{y}) - c_d + P'((M\hat{y} + N\hat{x}))\hat{x} \equiv 0$  and  $P(N\hat{x} + M\hat{y}) - c_f + t + P'((M\hat{y} + N\hat{x}))\hat{y} \equiv 0$ , this implies  $\hat{x}$  and  $\hat{y}$  is a continuously differentiable function of  $N$ .

Let  $\tilde{\pi}(N) = [P(N\hat{x} + M\hat{y}) - c]\hat{x}$  denote each domestic firm's profit in the stage 2 equilibrium. Denote the stage 1 profits as  $\pi(N) \equiv \tilde{\pi}(N) - K$ .

**Lemma 2:** In the equilibrium of the Stage 2 subgame, each domestic firm's stage 1 profit,  $\pi(N)$ , is strictly decreasing in  $N$  for all  $N > 1$ .

**Proof:** See Appendix.

We assume  $\pi(1) \geq 0$ , which is equivalent to  $K < \bar{K} \equiv \tilde{\pi}(1)$ . This condition guarantees that at least one firm will enter at stage one of the game. As before, we define the free-entry number of home firms to be the largest  $N \geq (1)$  such that  $\pi(N) = 0$ . Let  $N_f$  denote the free entry number of home firms.

The objective of the social planner is to maximize the home country's welfare, by controlling the number of domestic firms that enter the industry. We represent the home country's surplus as:

$$\int_0^{N\hat{x}+M\hat{y}} P(s)ds - Nc_d\hat{x} - NK - M(P(N\hat{x} + M\hat{y}))\hat{y} + tM\hat{y} \equiv W(N), \quad (6)$$

where  $\hat{x}$  and  $\hat{y}$  respectively denote equilibrium quantities produced by a representative domestic and foreign firm. The home government chooses  $N = N^*$ , such that  $N^*$  maximises

$W(N)$ . The solution  $N^*$  is implicitly obtained by setting  $W'(N)|_{N=N^*} = 0$ , where

$$W'(N) = \pi(N) + P(N\hat{x} + M\hat{y}) - c_d \left( N \frac{\partial \hat{x}}{\partial N} \right) - M\hat{y} \frac{\partial (P(N\hat{x} + M\hat{y}))}{\partial N} + tM \frac{\partial \hat{y}}{\partial N}.$$

Recall that  $\pi(N)$  is strictly decreasing in  $N$  for all  $N > 1$  (Lemma 2). It then follows that  $N_f < N^*$  holds if and only if  $\pi(N^*) < \pi(N_f) \equiv 0$ . Given  $W'(N)|_{N=N^*} = 0$ , we have the following implication.

**Proposition 3:** The number of home firms in the free-entry equilibrium is socially insufficient if and only if

$$\underbrace{(P(N\hat{x} + M\hat{y}) - c_d) \left( N \frac{\partial \hat{x}}{\partial N} \right)}_{\text{Business Stealing Effect}} - \underbrace{M\hat{y} \frac{\partial (P(N\hat{x} + M\hat{y}))}{\partial N}}_{\text{Transfer Effect}} + \underbrace{tM \frac{\partial \hat{y}}{\partial N}}_{\text{Lost Tariff Revenue}} > 0, \quad (7)$$

where all the expressions are evaluated at  $N = N^*$ .

The intuition of this result is similar to the voluntary export restraint (VER) case, when  $t = 0$ . Thus the first two terms of (7) do not require further elaboration. The third term,  $tM \frac{\partial \hat{y}}{\partial N}$ , captures the effect of entry on tariff revenue. As the number of firms increases, each foreign firm cuts back production. Thus,  $\frac{\partial \hat{y}}{\partial N} < 0$  which in turn implies that  $tM \frac{\partial \hat{y}}{\partial N} < 0$  as long as  $t > 0$ . This negative effect of entry on tariff revenue reinforces the ‘business stealing effect’ and increases the possibility of ‘socially insufficient entry’.

## 4.2 Linear Demand

In this subsection, we characterize the insufficient entry result for linear demand. Note that we have already verified that Assumption 1(i) holds for linear demand. Furthermore it is easy to verify that Assumption 3 holds for linear demand.

Stage 2 equilibrium quantities for domestic and foreign firms respectively are:

$$\hat{x} = \frac{a - (M + 1)c_d + M(c_f + t)}{(M + N + 1)}, \quad \hat{y} = \frac{-(N + 1)(c_f + t) + a + Nc_d}{(M + N + 1)}.$$

The equilibrium profit for the domestic and foreign firms respectively are:

$$\hat{\pi}_d = \left[ \frac{a - (M + 1)c_d + M(c_f + t)}{(M + N + 1)} \right]^2 = \hat{x}^2, \quad \hat{\pi}_f = \left[ \frac{-(N + 1)(c_f + t) + a + Nc_d}{(M + N + 1)} \right]^2 = \hat{y}^2.$$

Setting  $\hat{\pi}_d = K$  and solving for  $N$  gives the free entry equilibrium number of firms ( $N_f$ ):

$$N_f = \frac{a - c_d(M + 1) + M(c_f + t)}{\sqrt{K}} - M - 1.$$

To ensure that at least one home firm enters in the free entry equilibrium we assume that

$$K < \left[ \frac{a - (M + 1)c_d + M(c_f + t)}{M + 1} \right]^2 \equiv \bar{K}. \quad (8)$$

Since  $W(N)$  is strictly concave for  $N$ , insufficient entry occurs globally if and only if it occurs at the margin. Thus insufficient entry occurs if and only if the following holds:

$$\left. \frac{\partial W(N)}{\partial N} \right|_{N=N_f} \equiv - \frac{K((a - (2M + 1)c_d + 2M(c_f + \frac{3}{2}t)) - (2M + 1)\sqrt{K})}{(a - (M + 1)c_d - M(c_f + t))} > 0. \quad (9)$$

Rearranging the above condition we get

$$K > \left[ \frac{(a - c_d(2M + 1) - (c_f - \frac{3}{2}t)2M)}{(1 + 2M)} \right]^2 \equiv \tilde{K}. \quad (10)$$

Now we are ready to state the main result of this subsection.

**Proposition 4:** For linear demand insufficient entry occurs if and only if  $\tilde{K} < K < \bar{K}$  where  $\bar{K}$  and  $\tilde{K}$  as defined by (8) and (10). Furthermore,  $\tilde{K}$  is increasing in  $t$  implying that insufficient entry becomes more likely as trade cost declines.

The first part of Proposition 4 states that there exists parameterizations of  $K$  such that insufficient entry occurs. To understand the logic of the second part of the proposition we can decompose the effect of a domestic entrant into three effects. These three effect has been stated previously in (7), they are *business stealing effect*, *transfer effect* and the *lost tariff revenue*. These effects are respectively denoted by the first, second and third term on the right hand side of equation (7). First, consider the business stealing effect (denoted by  $(P(N\hat{x} + M\hat{y}) - c_d)(N\frac{\partial \hat{x}}{\partial N})$ ). As tariffs decreases, we find a reduction in the business stealing effect this can partially be explained by the reduction in the domestic share of output. Thus this increases the possibility of ‘insufficient entry’. Next, consider the transfer effect (denoted by  $-M\hat{y}\frac{\partial(P(N\hat{x}+M\hat{y}))}{\partial N}$ ), a reduction in tariff leads to an expansion of foreign production. This results in an a larger transfer effect, which works in favor of insufficient entry. Finally, consider the effect of lost tariff revenue (denoted by  $tM\frac{\partial \hat{y}}{\partial N}$ ). Naturally, as  $t$  shrinks the magnitude of decline tariff revenue also declines, which also works in favor of ‘insufficient entry’. Thus we find that a reduction in tariffs leads to a greater possibility of insufficient entry.

## 5 Endogenous Entry of Foreign Firms

Up to this point we have highlighted the possibility of insufficient entry by assuming an exogenous number of foreign firms. In this section, we allow foreign firms to freely enter into the domestic economy and consider whether ‘insufficient entry’ still holds.

Under the current homogenous goods framework we find that (except for special cases) domestic and foreign firms cannot coexist in equilibrium. Consider the case where domestic firms are more efficient ( $c_d < c_f$ ) and face lower entry costs ( $K_d < K_f$ ). Naturally, no foreign firm would produce a strictly positive amount. Analogously, if the foreign firms were more efficient ( $c_d > c_f$ ) and faced lower entry costs than their domestic counterparts ( $K_d > K_f$ ), then no domestic firm would produce a strictly positive amount. This might suggest that if  $c_i < c_j$  and  $K_i > K_j$ , where  $\{i, j\} \in \{d, f\}$ , and  $i \neq j$ , both domestic and foreign producers might produce positive quantities in the free entry equilibrium. We find that almost always only one type of firm (foreign or domestic) will produce a strictly positive amount in the free entry equilibrium. Thus to reexamine the possibility of insufficient entry under an open economy setup (i.e. existence of foreign firms) we need to introduce product differentiation. Note that in the presence of product differentiation insufficient entry can occur under a closed economy setting. The way we introduce product differentiation is slightly different to what is common in the literature. Particularly, all foreign firms and domestic firms produce a homogenous product  $Y$  and  $X$  respectively. However,  $X$  and  $Y$  are different products. This is in contrast with the standard modeling of differentiated products where all firms produce differentiated products.

### 5.1 Model

Suppose there are  $N$  domestic firms and  $M$  foreign firms. Let  $x_i$  represent the output produced by the  $i^{th}$  domestic firm and  $y_j$  denote the output produced by the  $j^{th}$  foreign firm. Define  $X = \sum_{i=1}^N x_i$  and  $Y = \sum_{j=1}^M y_j$ , as the aggregate domestic and foreign output. Consumers solve the following problem:

$$\max_{\{X, Y\}} U(X, Y) + Z \quad s.t. \quad P_X X + P_Y Y + Z \leq I,$$

where  $U(X, Y)$ ,  $P_X$ ,  $P_Y$ ,  $I$  and  $Z$  captures utility, domestic price, foreign price, income and the numéraire good. Following Horstmann and Markusen (1986), we have the following

utility specification:

$$U(X, Y) = a(X + Y) - \frac{1}{2}(X^2 + Y^2) - bXY,$$

which give rise to the following inverse demand functions:

$$P_X = a - \sum_{i=1}^N x_i - b \sum_{j=1}^M y_j, \quad P_Y = a - b \sum_{i=1}^N x_i - \sum_{j=1}^M y_j.$$

Observe that we have,  $\frac{\partial P_X}{\partial X} < 0$ ,  $\frac{\partial P_Y}{\partial Y} < 0$ ,  $\frac{\partial^2 P_X}{\partial X \partial Y} = \frac{\partial^2 P_Y}{\partial Y \partial X} < 0$ . Using these inverse demand function we have the following profit functions:

$$\pi_i = \left( a - \sum_{i=1}^N x_i - b \sum_{j=1}^M y_j - c_d \right) x_i, \quad i = 1, 2, \dots, N; \quad (11)$$

$$\pi_j = \left( a - b \sum_{i=1}^N x_i - \sum_{j=1}^M y_j - c_f - t \right) y_j, \quad j = 1, 2, \dots, M; \quad (12)$$

where  $\pi_i$  and  $\pi_j$  represent domestic and foreign profits respectively. Solving the profit maximization problem we obtain the following equilibrium outputs and profits:

$$\hat{x} = \frac{(M(1-b) + 1)a - (M+1)c_d + bM(c_f + t)}{N(M(1-b^2) + 1) + M + 1}; \quad \hat{\pi}_i = \hat{x}^2 = \pi_d(N, M); \quad \text{for all } i = 1 \dots N,$$

$$\hat{y} = \frac{(N(1-b) + 1)a - (N+1)(c_f + t) + bNc_d}{N(M(1-b^2) + 1) + M + 1}, \quad \hat{\pi}_j = \hat{y}^2 = \pi_f(N, M); \quad \text{for all } j = 1 \dots M.$$

Let  $N_f$  and  $M_f$  respectively denote the number of home and foreign firm in the free entry equilibrium. Then the pair  $(N_f, M_f)$  must satisfy the following:

$$\hat{\pi}_d(N, M) = \hat{x}^2 = K, \quad \hat{\pi}_f(N, M) = \hat{y}^2 = K.$$

We assume that  $\hat{\pi}_d(1, 1) > K \equiv \bar{K}$  and  $\hat{\pi}_f(1, 1) > K \equiv \hat{K}$ , to ensure that at least one foreign and domestic firm enters in the free entry equilibrium. Now consider the second best welfare problem. Given the utility function  $U(X, Y)$  and the inverse demand functions  $P_X$  and  $P_Y$ , we have the following welfare function:

$$W(N, M) = U(N\hat{x}, M\hat{y}) - P_Y(N\hat{x}, M\hat{y})M\hat{y} - c_d N\hat{x} + tM\hat{y} - NK.$$

Assuming concavity for  $W(N, M)$ , for insufficient entry to occur we require  $W'(N)|_{N=N_f, M=M_f} > 0$ . Specifically we find:

$$\left. \frac{dW(N)}{dN} \right|_{N=N_f, M=M_f} = (P_X - c_d)N \frac{d\hat{x}}{dN} - Y \frac{dP_Y}{dN} + t \frac{dY}{dN}. \quad (13)$$

Now consider the free trade case, i.e. let  $t = 0$ . Then (13) reduces to:

$$\left. \frac{dW(N)}{dN} \right|_{N=N_f, M=M_f} = -\hat{Y} \frac{\partial \hat{P}_Y}{\partial X} \hat{x}.$$

Since  $Y > 0$ ,  $\hat{x} > 0$ , and  $\frac{\partial P_Y}{\partial Y} < 0$ , we find that:

$$\left. \frac{dW(N)}{dN} \right|_{N=N_f, M=M_f} > 0, \quad (14)$$

which implies insufficient entry.

**Proposition 5:** Under free trade, the free entry number of domestic firm is always socially insufficient. More precisely (14) holds for all  $K < \min\{\bar{K}, \hat{K}\}$ .

## 6 Conclusion

In this paper we have presented a new perspective on the social desirability of free entry. It is well known that in a closed economy free entry leads to socially excessive number of firms. We showed that this result does not necessarily hold in the presence of foreign firms. In particular, we demonstrated that free entry might lead to socially insufficient number of firms under a wide range of parameterizations. Furthermore, under linear demand we also showed that trade liberalization increases the likelihood for insufficient entry. Several countries including Japan, South Korea and India pursued protectionist trade policies. On the domestic front entry regulation policies were put in place. It has often been argued that the “excess entry theorem,” can provide a justification for entry regulation as a way of improving social welfare. Our theoretical analysis suggests that while entry regulation of home firms at the margin improves welfare in a closed economy, it might actually reduce welfare in an open economy setting.

## Appendix

**Claim 1.** Assumption 1 implies that the system of equations (1) yields a unique solution  $x_1^* = x_2^* = \dots \equiv x^* > 0$ , which also constitutes the unique equilibrium of the stage 2 subgame.

**Proof.** Suppose that a solution to the systems of equations (1) exists, which is denoted by  $\{x_d\}_{d=1}^N$ , where  $x_d > 0$ , for all  $d$ . Rewriting (1) yields  $x_d = -(P(\sum_{d=1}^N x_d + Y) - c)/P'(\sum_{d=1}^N x_d + Y)$  for all  $d$ . Hence, if a solution exists, the solution is symmetric. Summing (1) for all  $N$  yields  $N(P(\sum_{d=1}^N x_d + Y) - c) + \sum_{d=1}^N x_d P'(\sum_{d=1}^N x_d + Y) = 0$ . Define  $f(Q) \equiv N(P(Q) - c) + X P'(Q)$ , where  $Q = \sum_{d=1}^N x_d + Y$  and  $X = \sum_{d=1}^N x_d$ . By Assumption 1(i) and  $P'(Q) < 0$ , we have (i)  $\lim_{Q \rightarrow 0} N(P_0 - c) > 0$  and (ii)  $\lim_{Q \rightarrow \infty} f(Q) < 0$  noting that  $\lim_{Q \rightarrow \infty} P(Q) \equiv P_\infty = 0$ . Furthermore, applying Assumption 1(ii) we have  $f'(Q) = (N + 1)P'(Q) + X P''(Q) < 0$ . Since  $f(Q)$  is continuous for all  $Q > 0$  (implying also for all  $X > 0$ ), there exists a unique value  $X^* (> 0)$ , such that  $N(P(X^* + Y) - c) + X^* P'(X^* + Y) = 0$ . This implies that the solution is given by  $x_1^* = x_2^* = \dots = x_N^* = X^*/N \equiv x^* (> 0)$ .

Now we show that this solution characterizes the unique equilibrium of the stage 2 subgame. Let  $\pi_d^D \equiv [P(Y + x_d + \sum_{i \neq d} x_i) - c]x_d$  and suppose  $\{x_d\}_{d=1}^N$  constitutes an equilibrium. Then the following condition must hold

$$\frac{\partial \pi_d^D}{\partial x_d} = P(Y + x_d + \sum_{i \neq d} x_i) - c + P'(Y + x_d + \sum_{i \neq d} x_i)x_d = (\leq)0, \text{ if } x_d > (=)0, \quad (\text{A.1})$$

for all  $d$ . First, suppose  $x_d = 0$  for all  $d$ . We then have  $\partial \pi_d^D / \partial x_d |_{(x_1, \dots, x_d) = (0, \dots, 0)} = P(Y) - c > 0$  for all  $d$ , but this contradicts (A.1). Second, suppose there exist  $k$  and  $l$ , such that  $x_k > x_l = 0$ . Then, from condition (A.1), we have  $P(Y + x_k + \sum_{i \neq k} x_i) - c = -P'(Y + x_k + \sum_{i \neq k} x_i)x_k > 0$  and  $P(Y + x_l + \sum_{i \neq l} x_i) - c + P'(Y + x_l + \sum_{i \neq l} x_l)x_l = P(Y) - c \leq 0$ . This is a contradiction. Hence, if  $\{x_d\}_{d=1}^N$  constitutes an equilibrium, then  $x_d > 0$  must hold for all  $d$ . Then the analysis in the previous paragraph implies that  $x_d = x^*$  for all  $d$  is the sole candidate for the equilibrium of the stage 2 subgame. Finally, we show that  $\partial^2 \pi_d^D / \partial x_d^2 < 0$ , which implies that  $x_d = x^*$  for all  $d$  indeed constitutes an equilibrium. We have that  $\left. \frac{\partial^2 \pi_d^D}{\partial x_d^2} \right|_{x_i = x^* \forall i \neq d} = 2P'(Y + x_d + (N - 1)x^*) + x_d P''(Y + x_d + (N - 1)x^*)$ . If  $P''(\cdot) < 0$ , trivially  $\left. \frac{\partial^2 \pi_d^D}{\partial x_d^2} \right|_{x_i = x^* \forall i \neq d} < 0$ . If  $P''(\cdot) > 0$ , we have  $x_d P''(\cdot) < (Q)P''(\cdot)$ , this implies  $2P'(\cdot) + x_d P''(\cdot) < 2P'(\cdot) + (Q)P''(\cdot)$ . By Assumption 1 (ii),  $2P'(\cdot) + (Q)P''(\cdot) < 0$ ,

which implies  $\left. \frac{\partial^2 \pi_d^D}{\partial x_d^2} \right|_{x_i=x^* \forall i \neq d} < 0$ . Thus the stage 2 subgame has a unique equilibrium in which each firm  $d$  chooses  $x_d = x^*$ . QED

**Proof of Lemma 1.** Define  $\tilde{\Pi}(N) \equiv N\tilde{\pi}(N) = (P(\hat{Q}) - c)\hat{X}$ , where  $\hat{Q} = Y + N\hat{x}$  and  $\hat{X} = N\hat{x}$ . Since  $\tilde{\Pi}'(N) = N\tilde{\pi}'(N) + \tilde{\pi}(N)$  and  $\tilde{\pi}(N) \geq 0$ , to prove  $\tilde{\pi}'(N) < 0$  it suffices to show that  $\tilde{\Pi}'(N) < 0$  for all  $N > 1$ . We have that:

$$\begin{aligned} \tilde{\Pi}'(N) &= (P(\hat{Q}) - c) \frac{d\hat{X}}{dN} + (P'(\hat{Q})\hat{X}) \frac{d\hat{Q}}{dN}, \\ &= (P(\hat{Q}) - c) \frac{d\hat{X}}{dN} + P'(\hat{Q})(\hat{x} + (N-1)\hat{x}) \frac{d\hat{Q}}{dN}, \\ &= \frac{(P(\hat{Q}) - c) \frac{d\hat{Q}}{dN}}{\frac{d\hat{X}}{dN}} + P'(\hat{Q})(\hat{x} + (N-1)\hat{x}) \frac{d\hat{Q}}{dN}, \\ &= (P(\hat{Q}) - c + P'(\hat{Q})\hat{x}) \frac{d\hat{Q}}{dN} + (N-1)\hat{x}P'(\hat{Q}) \frac{d\hat{Q}}{dN}, \\ &= (N-1)\hat{x}P'(\hat{Q}) \frac{d\hat{Q}}{dN}, \end{aligned}$$

where the third equation follows since  $\hat{Q} = Y + \hat{X}$ , thus we find  $d\hat{Q}/d\hat{X} = 1$ , therefore  $d\hat{X}/d\hat{N} = d\hat{Q}/d\hat{N}$ , which implies the second last equality holds. Finally, the last equality holds because  $(P(\hat{Q}) - c + P'(\hat{Q})\hat{x}) = 0$  for all  $N > 1$ . Since  $P'(\hat{Q}) < 0$  for all  $Q > 0$ , we have  $\tilde{\Pi}'(N) < 0$  for all  $N > 1 \Leftrightarrow d\hat{Q}/dN > 0$  for all  $N > 1$ . Below we prove  $d\hat{Q}/dN > 0$  for all  $N > 1$ .

Since  $P(\hat{Q}) - c + P'(\hat{Q})\hat{x} = 0$ , for all  $N > 1$ , we have:

$$N(P(\hat{Q}) - c) + P'(\hat{Q})\hat{X} = 0 \tag{A.2}$$

for all  $N > 1$ . Differentiating (A.2) with respect to  $N$ , and noting  $d\hat{X}/d\hat{N} = d\hat{Q}/d\hat{N}$ , we find

$$\frac{d\hat{Q}}{dN} = -\frac{P(\hat{Q}) - c}{(N+1)P'(\hat{Q}) + \hat{X}P''(\hat{Q})}. \tag{A.3}$$

Since  $P(\hat{Q}) - c > 0$  for all  $N > 1$ , and  $(N+1)P'(\hat{Q}) + \hat{X}P''(\hat{Q}) < 0$  (Assumption 1 (ii))<sup>2</sup> it follows that  $d\hat{Q}/dN > 0$  for all  $N > 1$ . QED

<sup>2</sup>Trivially, if  $P''(\hat{X}) < 0$ , then  $(N+1)P'(\hat{Q}) + \hat{X}P''(\hat{Q}) < 0$ . If  $P''(\hat{X}) > 0$ , then  $\hat{X}P''(\hat{X}) < \hat{Q}P''(\hat{Q})$ ,

**Claim 2.** There exists unique  $N_f(> 1)$  such that  $\pi(N_f) = 0$ .

**Proof.** We have that  $\pi(N)$ , is continuous for all  $N > 1$  and strictly decreasing in  $N$  for all  $N > 1$  and  $\pi(1) \geq 0$  (by Lemma 1 and Assumption 2). It suffices to show  $\lim_{N \rightarrow \infty} \tilde{\pi}(N) = 0$ , which implies  $\lim_{N \rightarrow \infty} \pi(N) = -K$ . From the proof of Lemma 1, we have,  $\tilde{\pi}(N) = \tilde{\Pi}(N)/N$  with  $\tilde{\Pi}(1)$  denoting monopoly profits, this implies  $\tilde{\Pi}(N)/N < \tilde{\Pi}(1)/N$ . Thus it follows that,  $0 \leq \tilde{\pi}(N)/N \leq \Pi(1)/N$ , taking the limits of the boundaries i.e.  $N \rightarrow \infty$  yields the following  $0 \leq \lim_{N \rightarrow \infty} \tilde{\pi}(N)/N \leq \lim_{N \rightarrow \infty} \tilde{\pi}(1)/N$ . We find that  $\lim_{N \rightarrow \infty} \tilde{\pi}(1)/N = 0$ . Following the squeeze theorem we find that  $\lim_{N \rightarrow \infty} \tilde{\pi}(N)/N = 0$ , implying  $\lim_{N \rightarrow \infty} \pi(N) = -K$ . QED

**Proof of Lemma 2.** Define  $\tilde{\pi}(N) = (P(\hat{Q}) - c_d)\hat{x}$ , where  $\hat{Q} = \hat{X} + \hat{Y}$ ,  $\hat{X} = N\hat{x}$  and  $\hat{Y} = M\hat{y}$ . We have that:

$$\frac{d\tilde{\pi}(N)}{dN} = P'(\hat{Q})\hat{x} \frac{d\hat{Q}}{dN} + (P(\hat{Q}) - c_d) \frac{d\hat{x}}{dN} = P'(\hat{Q})\hat{x} \left( \frac{d\hat{Q}}{dN} - \frac{d\hat{x}}{dN} \right), \quad (\text{A.4})$$

where the second equality holds since  $P(\hat{Q}) - c_d = -P'(\hat{Q})\hat{x}$ . To prove  $d\tilde{\pi}(N)/dN < 0$  it suffices to show that  $\frac{d\hat{Q}}{dN} - \frac{d\hat{x}}{dN} > 0$ , since  $P'(\hat{Q}) < 0$ .

First we show that  $d\hat{Q}/dN > 0$ . From the first order conditions (4) and (5) we have (i)  $P(\hat{Q}) - c_d + P'(\hat{Q})\hat{x} = 0$  and (ii)  $P(\hat{Q}) - c_f - t + P'(\hat{Q})\hat{y} = 0$ . Multiplying (i) and (ii) by  $N$  and  $M$  respectively and summing them we obtain

$$N(P(\hat{Q}) - c_d) + M(P(\hat{Q}) - c_f - t) + (M + N)P'(\hat{Q})\hat{Q} = 0. \quad (\text{A.5})$$

Differentiating (A.5) with respect to  $N$  gives

$$\frac{d\hat{Q}}{dN} = - \frac{P(\hat{Q}) - c_d}{(N + M + 1)P'(\hat{Q}) + \hat{Q}P''(\hat{Q})}. \quad (\text{A.6})$$

Since  $P(\hat{Q}) - c_d > 0$  for all  $N > 1$  and  $(N + M + 1)P'(\hat{Q}) + \hat{Q}P''(\hat{Q}) < 0$  (by Assumption 1 (ii)), it follows that  $d\hat{Q}/dN > 0$  for all  $N > 1$ .

Rearranging  $P(\hat{Q}) - c_d + P'(\hat{Q})\hat{x} = 0$  gives  $\hat{x} = -(P(\hat{Q}) - c_d)/P'(\hat{Q})$ . Differentiating  $\hat{x}$  with respect to  $N$  and simplifying we get,

which implies  $2P'(\hat{Q}) + \hat{X}P''(\hat{Q}) < 2P'(\hat{Q}) + \hat{Q}P''(\hat{Q})$ . By Assumption 1 (ii) it follows that  $2P'(\hat{Q}) + \hat{Q}P''(\hat{Q}) < 0$ , thus  $2P'(\hat{Q}) + \hat{X}P''(\hat{Q}) < 0$ .

$$\frac{d\hat{x}}{dN} = - \left( 1 + \frac{\hat{x}}{\hat{Q}} \cdot \frac{P''(\hat{Q})\hat{Q}}{P'(\hat{Q})} \right) \frac{d\hat{Q}}{dN}, \quad (\text{A.7})$$

which implies:

$$\frac{d\tilde{\pi}(N)}{dN} = P'(\hat{Q}) \left[ \frac{d\hat{Q}}{dN} \right] \left[ 2 + \frac{\hat{x}}{\hat{Q}} \cdot \frac{P''(\hat{Q})\hat{Q}}{P'(\hat{Q})} \right]. \quad (\text{A.8})$$

We have that  $\left[ 2 + \frac{\hat{x}}{\hat{Q}} \cdot \frac{P''(\hat{Q})\hat{Q}}{P'(\hat{Q})} \right] = \left[ \frac{\hat{Q}(2P'(\hat{Q}) + \hat{x}P''(\hat{Q}))}{\hat{Q}P'(\hat{Q})} \right]$ . Since,  $P'(Q) < 0$  and  $(2P'(\hat{Q}) + \hat{x}P''(\hat{Q})) < 0$  (this condition follows by applying Assumption 1 (ii)), this implies  $\left[ 2 + \frac{\hat{x}}{\hat{Q}} \cdot \frac{P''(\hat{Q})\hat{Q}}{P'(\hat{Q})} \right] > 0$ . Together with  $P'(Q) < 0$  and  $d\hat{Q}/dN > 0$  this implies  $\frac{d\tilde{\pi}(N)}{dN} < 0$ . QED

## References

- [1] Brenton, P.A. and Winters, L.A. “Voluntary Export Restraints and Rationing U.K. Leather footwear Imports from Eastern Europe.” *Journal of International Economics*, Vol. 34 (1993), pp.289-308.
- [2] Dean, J.M. and Gangopadhyay, S. “Market Equilibrium under the ‘threat’ of a VER.” *Journal of International Economics*, Vol. 30 (1991), pp.137-152.
- [3] Dixit, A.K. and Stiglitz, J.E. “Monopolistic Competition and Optimum Product Diversity.” *American Economic Review*, Vol. 67 (1977), pp.297-308.
- [4] Ghosh, A. and Morita, H. “Free Entry and Social Efficiency Under Vertical Oligopoly” *RAND Journal of Economics*, Vol. 38 (2007), pp.541-554.
- [5] Hamilton, C.B. “European Community External Protection and 1992 Voluntary Export Restraints applied to Pacific Asia.” *European Economic Review*, Vol. 35 (1991), pp.78-387.
- [6] Horstmann, I.J. and Markusen, J.R. “Up the Average Cost Curve: Inefficient Entry and the New Protectionism.” *Journal of International Economics*, Vol. 20 (1986), pp.225-247.
- [7] Ishikawa, J. “Who Benefits from Voluntary Export Restraints?.” *Review of International Economics*, Vol. 6 (1998), pp.129-141.
- [8] Mankiw, N.G. and Whinston, M.D. “Free Entry and Social Inefficiency.” *RAND Journal of Economics*, Vol. 17 (1986), pp.48-58.
- [9] Perry, M.K. “Scale Economies, Imperfect Competition, and Public Policy.” *Journal of Industrial Economics*, Vol. 32 (1984), pp.313-333
- [10] Song, E.Y. “Voluntary Export Restraints and Strategic Technology Transfers.” *Journal of International Economics*, Vol. 40 (1996), pp.165-186.
- [11] Spence, A.M. “Product Selection, Fixed Costs, and Monopolistic Competition.” *Review of Economic Studies*, Vol. 43 (1976), pp.217-235.

- [12] Suzumura, K. and Kiyono, K. "Entry Barriers and Economic Welfare." *Review of Economic Studies*, Vol. 54 (1987), pp.157-167
- [13] Vives, X. *Oligopoly Pricing: Old Ideas and New Tools*, MA: MIT Press, 1999.
- [14] von Weizsacker, C.C. "A Welfare Analysis of Barriers to Entry." *Bell Journal of Economics*, Vol. 11 (1980), pp.399-420.