ON THE PROBABILITY OF HELP*

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Abstract: The experimental and field studies surveyed by Latane and Nida (1981) establish an inverse relationship between the probability that a person is helped and the size of the group of potential helpers. Harrington (2001) attempts to account for this phenomenon using a 'rational choice' model in which agents play Nash strategies. In Harrington's model the probability that anyone helps a person in trouble decreases as the number of potential helpers increases. Also, the probability that a victim receives help is bounded below and away from zero. This second implication of the model in is somewhat at variance with the analysis of Hochman and Rogers (1969), Bergstrom (1970), Nakayama (1980) and Arrow (1981) where, if there are two rich people (potential helpers) and one poor person (victim), then 'helping' has the characteristics of a pure public good, with predictable consequences for the equilibrium level of help offered. The principle purpose of this paper is to extend the model of Harrington (2001) to include the Hochman and Rogers – Bergstrom – Nakayama – Arrow result as a special case.

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1. INTRODUCTION

The experimental and field studies reported by Latane and Nida (1981) establish an inverse relationship between the probability that a victim will receive help and the size of the group of potential helpers. Directly motivated by this work, Harrington (2001) proposes a 'rational choice' model which attempts to account for the following two phenomena: (A) the likelihood that a person will help another who is in trouble declines with an increase in the size of the group to which the potential helper belongs; and (B) the probability that anyone helps a person in trouble decreases as the number of potential helpers increases, but is generally bounded away from zero no matter how large the group of potential helpers becomes.

Outcome B is somewhat at variance with the results obtained by Hochman and Rogers (1969), Bergstrom (1970), Nakayama (1980) and Arrow (1981). As Hammond (1987) points out, in these models if the economy consists of just two people, one rich ('potential helper') and one poor ('person in trouble') and if the welfare of the rich depends on their income and also on the utility level of the poor, then depending on the details of the rich persons utility function, it might be individually rational for the rich person to transfer income to the poor person. However, if there are now two rich people and one poor person, there is scope for each potential helper to 'free-ride' on the help that the other might give. The Hochman and Rogers – Bergstrom – Nakayama – Arrow analysis is particularly interesting because it suggests that 'help' (specifically voluntary income transfers) have the characteristics of a classic public good. Consequently the Nash equilibrium of such a game will result in zero help as soon as the number of potential helpers is two or more[1]. Motivated by this observation we extend the model in Harrington (2001) and show how the extended model can produce the results of Hochman and Rogers – Bergstrom – Nakayama – Arrow as a special case. In particular we examine the preference structure that leads to Harrington's result that the probability of help is bounded away from zero, no matter how large the group of potential helpers becomes. To achieve our ends the paper is organised as follows: Section two presents the model and main argument while Section 3 presents some conclusions.

2. THE MODEL

We begin by sketching the model suggested by Harrington (2001) in which there are N potential helpers (agents)[2] and U_i is the utility function of agent i. Let H_i denote the event 'i helps another person' and let $\neg H_i$ denote the event 'i does not help anyone'. Let $H_{N/i}$ denote the event 'somebody other than i helps another person' and let $\neg H_N$ denote the event 'nobody helps another person'. For simplicity let:

$U_i(H_i \wedge \neg H_{N/i}) = a$, i.e. utility of i when they help and nobody else helps is 'a';

$U_i(\neg H_i \wedge H_{N/i}) = b$, i.e. utility of i when they don't help and somebody else helps is 'b';

$U_i(H_i \wedge H_{N/i}) = c$, i.e. utility of i when they help and somebody else also helps is 'c';

$U_i(\neg H_N) = d$, i.e. utility of i when nobody helps is 'd';

[1] The analysis also has the important public policy implication that if transfers to the poor are a public good then as Hammond (1987) notes: "... this is a *prima facie* argument for public intervention to redistribute income." Hammond (1987; p. 85)[1]. If on the other hand there is a non-zero probability that someone will make a voluntary redistribution then the case for public intervention is weakened.

[2] With slight abuse of notation 'N' will also denote the set of agents in the economy.

Definition (Nash equilibrium): A *Nash equilibrium in the helping game* is defined by N strategies, one for each player, in which each player's strategy maximizers their payoff given the strategies adopted by other players.

The outcome of the game, in particular the existence of a Nash equilibrium, will depend on the structure of the preferences of the agents in the group of potential helpers. Harrington (2001; p. 391) shows that if a > d and b > c then there are N pure strategy Nash equilibria in this game, each involving one player choosing H and the other N-1 players choosing ¬H[3]. This structure on preferences amounts to supposing that: (i) the utility that i receives from helping when nobody else does is greater than the utility they get when nobody helps so that $U_i(H_i \wedge \neg H_{N/i}) > U_i(\neg H_N)$ and a > d; and (ii) the utility i gets when they do not help but somebody else does help is greater than the utility they get when they help and somebody else helps so that $U_i(\neg H_i \wedge H_{N/i}) > U_i(H_i \wedge H_{N/i})$ and b > c. Harrington (2001) makes just this assumption about preferences.

__Assumption PH__: The preferences of everyone in the group of potential helpers is such that a > d and b > c.

Since the game is symmetric, the symmetric Nash equilibria necessarily involve players using mixed strategies. A mixed strategy involves randomisation which is represented by

[3] Harrington's proof is as follows: suppose that one player chooses H and the other N-1 players choose ¬H. Consider one of the non-helpers. If they choose ¬H then they get a payoff b while if they choose H they get a lower payoff c. Thus choosing ¬H is optimal. The player who chooses H gets a which exceeds the payoff d from choosing ¬H.

the probability of choosing H. Let p be the probability of agent i choosing H and (1 – p) be the probability of them choosing ¬H. Then i is indifferent between the actions H and ¬H if the expected utilities of the two actions are the same[4]. This occurs if:

$$(1-p)^{N-1}a + [1 - (1-p)^{N-1}]c = (1-p)^{N-1}d + [1 - (1-p)^{N-1}]b \quad (1)$$

Expanding (1) and collecting terms we get:

$$(1-p)^{N-1}(a-c) + (1-p)^{N-1}(b-d) = (b-c) \Leftrightarrow (1-p)^{N-1}[(a-c)+(b-d)] = (b-c) \quad (2)$$

$$\Rightarrow (1-p)^{N-1} = (b-c) / [(a-d)+(b-c)] \Rightarrow (1-p) = \{(b-c) / [(a-d)+(b-c)]\}^{1/(N-1)}$$

$$\therefore p^* = 1 - \{(b-c) / [(a-d)+(b-c)]\}^{1/(N-1)} \quad (3)$$

If Assumption PH holds then the term $\{(b-c)/[(a-d)+(b-c)]\} > 0$ and from (3) we see that as the size of the economy increases, i.e. as $N \to \infty$, $1/(N-1) \to 0$. Therefore:

$$\therefore \lim_{N\to\infty} p^* = \lim_{N\to\infty} 1 - \{(b-c) / [(a-d)+(b-c)]\}^{1/(N-1)} = 0 \quad (4)$$

[4] $(1-p)^{N-1}U_i(H_i \wedge \neg H_{N\backslash i}) + [1-(1-p)^{N-1}]U_i(\neg H_i \wedge H_{N\backslash i}) = (1-p)^{N-1} U_i(\neg H_N) + [1 - (1-p)^{N-1}] U_i(\neg H_i \wedge H_{N\backslash i})$

Thus the probability that a particular agent will help, declines as the size of the group of potential helpers grows. This rationalises observation (A).

What about the probability that *somebody* helps? Denote the equilibrium probability that at least one person helps by Q(N). By definition:

$$Q(N) = 1 - (1 - p^*)^N = 1 - \{(b - c) / [(a - d) + (b - c)]\}^{N/(N-1)} \qquad (5)$$

We now study the probability that nobody helps as the size of the economy grows. This probability is given by:

$$1 - Q(N) = \{(b - c) / [(a - d) + (b - c)]\}^{N/(N-1)} \qquad (6)$$

$$\therefore\ ln\,(1 - Q(N)) = N/(N-1) ln\{(b - c) / [(a - d) + (b - c)]\} \qquad (7)$$

Since $\partial ln\,(1 - Q(N))/\partial N = \partial(ln\,(1 - Q(N))/\partial(1 - Q(N)).\partial(1 - Q(N))/\partial Q(N)).\ \partial Q(N)/\partial N$

$$1/[1 - Q(N)].(-1).\ \partial Q(N)/\partial N = \partial\{(N/N - 1)\ ln\{(b - c) / [(a - d) + (b - c)]\}/\partial N \qquad (8)$$

The RHS of (8) yields:

$$[1/(N - 1) - N/(N - 1)^2].ln\{(b - c) / [(a - d) + (b - c)]\}$$

$$= [(N - 1)/(N - 1)^2 - N/(N - 1)^2].ln\{(b - c) / [(a - d) + (b - c)]\}$$

$$= -1/(N - 1)^2 ln\{(b - c) / [(a - d) + (b - c)]\}$$

$$\therefore 1/[1 - Q(N)].(-1).\ \partial Q(N)/\partial N = -1/(N-1)^2 \mathit{ln}\{(b-c) / [(a-d) + (b-c)]\} \tag{9}$$

$$\Rightarrow 1/[1 - Q(N)].\partial Q(N)/\partial N = 1/(N-1)^2 \mathit{ln}\{(b-c) / [(a-d) + (b-c)]\}$$

$$\Rightarrow \partial Q(N)/\partial N = [1 - Q(N)].\ \ln\{(b-c) / [(a-d) + (b-c)]\} \Rightarrow$$

$$\partial Q(N)/\partial N = 1/(N-1)^2\{(b-c)/[(a-d) + (b-c)]\}^{N/N-1} \mathit{ln}\{(b-c) / [(a-d) + (b-c)]\} \tag{10}$$

Provided $\{(b-c) / [(a-d) + (b-c)\} < 1$, (which is guaranteed by Assumption PH), then $\mathit{ln}\{(b-c) / [(a-d) + (b-c)]\} < 0$ and from (9), $\partial Q(N)/\partial N < 0$. Thus the addition of more potential helpers to the economy lowers the probability that anyone helps.

However, since $\partial Q(N)/\partial N < 0$ and $\lim_{N\to\infty} Q(N) = (a-d) / [(a-d) + (b-c)]$ we have that $Q(N) > (a-d) / [(a-d) + (b-c)]$ for all N, so there is a lower bound on the probability that someone helps, (see Harrington (2001) for further discussion).

As Hammond (1987) points out however, in the models of Hochman and Rogers (1969), Nakayama (1980) and Arrow (1981), 'helping' (in the form of endowment or income transfer), *might* happen in a two person economy where there is one potential helper (high income person) and one victim (low income person), but as soon as the number of helpers gets to two or more, 'helping' becomes a classic public good and the Nash equilibrium amount of help (redistribution) goes to zero with probability one. This is at variance with the predictions of Harrington (2001) where the equilibrium probability that at least one person helps is bounded below by $(a-d) / [(a-d) + (b-c)]$. Can the results of these two groups of models be reconciled?

2.1 Alternative preferences

As we've seen Harrington (2001) assumes that the utility an agent enjoys if they help and nobody else helps ('a') is greater than the utility they receive if nobody at all helps ('d'). From an economic and psychological point of view this is not obviously the case. For instance, if an agent helps and nobody else helps they may feel 'put upon' or exploited by the rest of the group of potential helpers. Consequently the positive benefit derived from the altruistic act of helping may be partially or totally negated by the disutility coming from the sense of being 'used' by the rest of the group of potential helpers. If this is the case then the value a may approach d. We now explore this consequences of this possibility by generalising Harrington's assumption that $a > d$ to allow $a \geq d$.

Assumption PH': *The preferences of everyone in the group of potential helpers are such that $a \geq d$ and $b > c$.*

Using Assumption B we may argue as follows. Let $a = d + \varepsilon$ with $\varepsilon \geq 0$. By substitution in (4) we have that:

$$Q(N) = 1 - \{(b - c) / [(\varepsilon) + (b - c)]\}^{N/(N-1)} \qquad (11)$$

As $\varepsilon \to 0$ in (11), $Q(N) \to 0$ independent of N, provided only that $N \geq 2$.

Also, $\partial Q(N)/ \partial N = 1/(N - 1)^2.\{(b - c)/[(\varepsilon) + (b - c)]\}^{N/N-1}.\ ln\{(b - c) / [(\varepsilon) + (b - c)]\}$ and as $\varepsilon \to 0$ this term is dominated by $ln\{(b - c) / [(\varepsilon) + (b - c)]\} \to 0$.

Therefore in the case where a = d adding another person to the group of potential helpers does not change the probability that somebody helps as that probability is already zero.

By extending the model in Harrington (2001) to allow $a \geq d$ we have allowed for the (realistic?) possibility that agents may resent being the only one to help in a group of potential helpers. In the limit, if this resentment makes agents indifferent between being the only one to help and seeing the event where the victim is not helped at all, then like the models of Hochman and Rogers (1969), Nakayama (1980) and Arrow (1981) this extended version of Harrington (2001) also predicts zero help when the group of potential helpers has two or more members.

3. Conclusion

The model of Harrington (2001) provides an interesting way to rationalise the twin empirical observations that the probability of a particular individual helping and the probability that a victim will receive any help at all, decreases with the size of the group of potential helpers.

In this paper we have shown that an extension of Harrington (2001) allows the model to predict the possibility of a rapid drop in the probability of help as soon as the number of potential helpers is two or more. Further, our analysis shows that there is no longer a non-zero lower bound on the probability that someone will help, a result consistent with results obtained in earlier economics literature particularly that due to Hochman and Rogers (1969), Bergstrom (1970), Nakayama (1980) and Arrow (1981).

REFERENCES

Arrow, K. J., (1981), "Optimal and voluntary income distribution", (in) *Economic Welfare and the Economics of Soviet Socialism: Essays in Honour of Abraham Bergson*, ed. S. Rosefield, Cambridge: Cambridge University Press, 267-288.

Bergstrom, T. C., (1970), "A 'Scandinavian consensus' solution for efficient income distribution among nonmalovalent consumers", *Journal of Economic Theory*, 2, 383-398.

Hammond, P. J., (1987), "Altruism", (in) *The New Palgrave Dictionary of Economics*, ed. J. Eatwell, M. Milgate and P. Newman, London: Macmillan Press.

Harrington, J. E., (2001), "A simple game-theoretic explanation for the relationship between group size and helping", *Journal of Mathematical Psychology*, 45, 389-392. doci:10.1006/jmps.1999.1313.

Hochman, H. M. and Rogers J. D., (1969), "Pareto optimal redistribution", *American Economic Review*, 59, 542-557.

Latane, B. and Nida, S., (1981), "Ten years of research on group size and helping", *Psychological Bulletin*, 89, 308-324.

Nakayama, M., (1980), "Nash equilibria and Pareto optimal income redistribution", *Econometrica*, 48, 1257-1263.

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