A General Framework for Analyzing Water Property Rights

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A General Framework for Analysing Water Property Rights

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Abstract

Significant imbalances in water allocation between the irrigation sector and other sectors have motivated the introduction of water markets in many regions with the hope of transferring water with low value in irrigation to higher valued uses. However, for the markets to work, water property rights must be well defined. In this paper, a general framework is provided for describing and economically analysing a variety of water allocation systems as used (or proposed) in Australia and elsewhere. It is shown that centralised systems are generally inefficient, but the inefficiency decreases as the storage limit allocated to each water user increases and as the interference into water storage by the central authority decreases. It is also shown that conversion of water rights between classes to reduce water allocation inefficiency in these systems can change the identity of water rights and impose external costs on third parties. These weaknesses of centralised systems can be addressed by introducing decentralised systems. However, the large number of transactions in the spot markets under decentralised systems may make decentralised systems undesirable. Introduction of two classes of inflow rights, as suggested in this paper, in decentralised systems will help to reduce transaction costs.

Keywords: Water storage, water rights, water allocation, decentralised storage systems.
Introduction

Designs of water allocation system play an important role in the management of water supply risk. Irrigation water commodity with its special characteristics including high physical volume to price ratios of water, high seepage and evaporation losses involved with water storage in practical farm dams, negative externality of salinity and water logging caused by on-farm water storage and high costs of land needed to build farm dams is often too costly to store on farm. Conversely, the cost of storage is much lower at central dams due to the low surface to volume ratios of central dams (so that the evaporation losses are low) and to the fact that water seepage from central dams may contribute to flows downstream and generate positive external effects. With storage being a key element bridging water supply between seasons, water supply risk management relies significantly on the design of water allocation systems that facilitates efficient storage at central storage facilities.

In constructing a water allocation system, it is important to note that in many regions, markets in which water rights (i.e. rights to water in the current and future years) are traded (often called “permanent” transfer) and markets in which water allocations in the current year are traded (often called “temporary” transfer of water) have been established. It is essential that the implemented water allocation system facilitates efficient operations of these markets. For these water markets to improve the allocative efficiency within the irrigation sector (by moving water from low-valued activities to higher-valued activities), water property rights must be well defined. Because a commodity is defined by its location, the time when it can be consumed and conditions under which it can be consumed, the allocative efficiency of a water market also depends on the property right to water delivery, the property right to water storage infrastructure and the property right to the by-products that water
consumption may generate [Heaney et al., 2006]. Theoretically, all property rights can be explicitly defined and well enforced. However, the establishment of a new property system is costly and extra monitoring activities incur on-going costs. Judgements of actions or inaction in relation to a perceived problem of ill-defined property rights require an understanding of resource allocation impacts beyond the simple marginal values of water.

In irrigation schemes, the property right to water delivery and to water consumption by-products has often been clearly defined while the property right to water storage infrastructure is often less clear. Although several decentralized forms of allocation of storage capacity have emerged, in most systems water storage remains under the control of a central authority. A concern regarding the transition from a centralised system to a decentralised system in which water storage is fully controlled by water users is that the transaction costs may be higher in a decentralised system. As Freebairn and Quiggin [2006] have shown, the number of transactions is lower in a centralised system that has more water right classes. Since existing decentralised systems commonly have only one class of water right, the number of transactions and the transaction costs may be higher than in a centralised system.

In this paper, the efficiency of water allocation systems is analysed with a view to suggesting possible improvements on existing systems. Water allocation institutions are reviewed and generalised in the first section. Australia, with its relatively dry climate and significant irrigated agricultural sector, and with a federal governance structure assigning power over water to separate states rather than the national government, has experience with a variety of water allocation systems. While the water allocation institutions described are Australian, they are more generally applicable. In the second section, the impacts of structural changes on water right
identity and the impacts of various components of the system on allocative efficiency are analysed. A discussion about possible improvements for existing systems is provided in the third section.

1. Review of water allocation institutions

Water allocation systems can be classified as centralised or decentralised systems [Dudley, 1990]. In a centralised system, the central authority fully or partially determines the amount of water saved or stored for future release. In a decentralised system, water storage is fully determined by water users. Examples of centralised systems include the so called “priority sharing” and “proportional sharing” systems. Examples of decentralised systems include “capacity sharing” and “continuous accounting” systems. No unanimous agreement exists on the best arrangement. Priority sharing has been implemented in several states in Australia (New South Wales (NSW), Queensland and newly adopted in Victoria) and in western states (California, Colorado) in the United States. Proportional sharing has been implemented in Chile and was previously implemented in Victoria. Capacity sharing has been implemented in the St George irrigation area and MacIntyre Brook in southern Queensland. Continuous accounting has been implemented in the Border Rivers, Gwydir and Namoi regions in NSW [Hughes, 2009; PC, 2003].

All the systems described here apply to a scenario in which water is collected (the “inflow”) during a period (typically a year or season, but in one case a day) either for use in that same period or alternatively for carrying-over (“storage”) to the following period. Water rights can relate to the inflows and/or to the storage between periods.
1.1. Water reliability

The reliability of water supply to a water right is a key element in the commonly accepted definition of water rights in centralised systems. It has been defined as ‘the frequency with which full seasonal allocations are expected to be available’, e.g. see DSE [2011] or Alaouze [1991a]. Defining water right reliability as the probability that the right is fully allocated is useful for water rights that frequently receive full allocation. For water rights that frequently receive less than full allocation, water users are more interested in knowing the probability distribution of water allocation below full allocation. In this case, the notion of stochastic dominance (first degree) is useful in characterising an increase or a decrease in water supply reliability. For example, a reform may be said to increase the reliability of a water right if the distribution of water allocation to the right after the reform stochastically dominates the distribution of water allocation to the right before the reform. The analysis below will employ this broader definition of changes in water supply reliability when appropriate.

1.2 Priority sharing

In priority sharing systems, water rights are categorised according to their priority ranks. Each water right of each kind is assigned with a nominal volume. Water inflow to the dam and the central carryover are shared in a priority order. Available water is allocated to a particular category of water rights only when all water rights of higher priority have been fully allocated, that is after they have received allocations equal to their nominal volumes [Brennan and Scoccimarro, 1999].

This method of sharing water among water rights is similar to the way a borrowing country allocates its wealth to service debts from different lenders or the way a
company allocates its returns to lenders and creditors. In the context of country debt, a country’s debt servicing capacity is stochastic and when the debt service capacity is high, all creditors can collect their interest income, but when the debt service capacity is low, senior creditors such as the International Monetary Fund (IMF) and World Bank are paid first and any residual is used to pay junior private lenders (commercial banks) [Bartolini and Dixit, 1991]. The analogy is, however, not perfect since in the debt case, payments that have not been made in one year will still have to be made in the future, while in the water case, any lack of allocation of inflow in one year to a water right holder is not made up in the following year.

Depending on the quantities of water rights of each kind, a priority system can mean a rapid drop in the reliability of water rights below the highest priority category and potentially a high probability that lower priority right holders will receive no water. One way to give lower right holders some opportunity for accessing water in a priority system is through allocating dam space to each water right holder so that water allocated in high availability years can be privately carried over to the following year. The amount of dam space allocated to each category of water right may be different. Often, together with the dam space allocated, there is also a rule to limit the impacts of one water right holder’s storage on other water right holders’ water allocation.

As an example of a priority sharing system, the system implemented in NSW has two categories of water rights, called high security water rights and low (or general) security water rights, respectively. Because the aggregate number of high security water rights is in fact relatively low relative to the probability distribution of inflows, high security water rights receive full allocations in almost all years. The lower priority general security water rights are fully allocated only 37% of the time. High
security water right holders are not allowed to carry over water to the following year while general security water right holders can carry over water up to some limit [DIPNR, 2005]. For general security water rights, the sum of private carryover and announced allocation (for the current, not-yet-completed, period) cannot exceed the nominal volume for each water right.

1.3 Proportional sharing

Proportional sharing is the limiting case of priority sharing in which the number of water right categories reduces to one. In this system, all water rights are the same and they share the water inflow and the central carryover proportionally. Part of the dam space may be allocated to water right holders for private inter-period carryover [Howe et al., 1986; PC, 2003].

In the proportional sharing system implemented in Victoria, right holders could choose the time(s) within the current year to consume their allocated water, but could not save their water to the following year. Unused water would be returned to the common carryover pool. Water storage was determined according to a central policy.

1.4 Capacity sharing

The concept of capacity sharing was introduced by Dudley and Musgrave [1988]. In this system, each water user is provided with a share of water inflow and a share of dam space. If a market for dam space is established, users may change their share of dam space by trading with other users. Similarly, users can change their inflow shares if a market for inflow shares is available. Water users decide how much water to use in the current “accounting” period and how much water to carry over to the following accounting period. An accounting period may be a month, a quarter or a year. Water accounting occurs at the end of each period and users who have their dam share full at
that time will lose their inflow to the common pool. Usage of dam space within a period is not accounted for. For example, a water user keeping his dam space full until near the end of the period and then using all his stored water before the end of the period will not lose any water inflow allocated for the period if his share of water inflow is less than his dam space.

1.5 Continuous accounting

In this system, each water user is provided with a nominal volume of dam space. Water inflow is continuously (i.e. daily) allocated to water users. The amount of water inflow a water user receives is proportional to the nominal volume of his dam space. Unused water is automatically carried over to the next day until the user’s accumulation balance reaches his nominal volume [Hughes, 2009].

The continuous accounting system is fundamentally different from the other systems since it continuously restricts water users to use their nominal dam space. A water user cannot fill his dam space with water and rely on other users’ dam space for storing new water within a season as in other systems. In addition, a continuous accounting system is also different from a capacity sharing system in that the right to water inflow is not separated from the right to dam space, though it would be possible to define a continuous accounting system in which the two types of rights are separated. In that case, the difference between a capacity sharing system and a continuous sharing system would be the length of the accounting period, i.e. a day versus a month or longer period.

1.6 Generalization of water allocation systems

The above water allocation systems can be described by five variables (several of which are vector valued): the number of water right categories, the number of water
rights in each category, the limit imposed on carryover for each category of water right, the central storage rule and the length of an accounting period.

A general system has $J$ classes of water rights, where each class $j$ has $N_j$ water rights. Water rights in class $j$ have a priority ranking $j$ and without loss of generality, it can be assumed that class $j$ water rights are senior to class $k$ water rights if $j < k$. Each water right has a nominal volume $V$ and the total amount of water assigned to each water right must be lower than this nominal volume at all times\(^2\). Furthermore, each water right in class $j$ has a carryover limit $C_j \geq 0$ and the carryover limit must be lower than the nominal volume, $C_j \leq V$ for otherwise, private carryover may result in a shortage of storage space for central carryover\(^3\).

When an amount $Y_j$ is announced to be allocated to water right $j$, a holder of water right $j$ may not be able to receive the entire amount $Y_j$ if the sum of his private carryover from the previous period, $S_j$, and the new allocation $Y_j$ exceeds the nominal volume $V$. In this case, the water right holder can only receive a part of water allocation so that the total amount of water he has is equal to the nominal volume. The amount of water that a holder of a water right $j$ receives, $X_j$, can be expressed as:

$$X_j = \min[Y_j + S_j, V] - S_j.$$ 

Private water carryover $S_j$ in each period is constrained by the limit $C_j$:

\(^2\) In a continuous accounting system, $V = K/\sum N_j$ and in other systems, $V = 1ML$. 

\(^3\) This is not a problem if no water is imported into the system since the total amount available for carryover is less than the dam capacity. When water import is allowed, the constraint limits the total carryover to be below the dam capacity.
\[ S_j \leq C_j. \]

The total amount of water announced to be allocated but not actually received and taken up by water right holders due to the constraint is \( Y - \sum_{j=1}^{J'} X_j \), where \( Y \) is the total announced allocation. This water is called ‘internal spillage’ by Dudley and Musgrave (1988). The internal spillage, public water storage (net of evaporation and seepage losses) and new water inflow (net of external spillage) will form the total public water availability for allocation in the next period\(^4\). The central storage in the system is described by a rule \( S_c(A) \) that depends on the total public water availability \( A \). This planned storage amount is equal to the difference between the total public water availability and the total announced allocation, \( S_c(A) = A - Y \).

The general system reduces to a priority system in NSW when \( J = 2, V = 1\text{ML}, C_1 = 0 \) and \( C_2 > 0 \) and to a proportional sharing system in Victoria when \( J = 1, V = 1\text{ML}, C = 0 \). A continuous accounting system is generated when \( J = 1, V = K / \sum_{j=1}^{J} N_j, S_c = 0 \) and \( C = V \). A capacity sharing system without trading in inflow share can be generated in the same way as for a continuous accounting system. The above general system, however, cannot generate a capacity sharing system with traded inflow shares because the internal spillage rule is ‘water right based’, while in a capacity sharing system with traded inflow shares, the internal spillage rule is necessarily ‘water user

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\(^4\) For example, when irrigation occurs in only one season in a year and the accounting period is one year, if we are currently in a period say 2011, the authority has some planned public water storage from 2011 into 2012. As well, there will be internal spillage occurring in 2011, which will also carry over to 2012. Water inflow collected between the end of the current irrigation season and the end of the next season will be counted as the inflow used for 2012 season.
based’, which depends on the number of capacity rights and the number of water inflow rights held by each water user.

In a capacity sharing system with traded inflow shares, the carryover limits must be the same for all water right classes for which water inflow rights are traded. Otherwise, a switch of water inflow rights between two water rights of two different classes will lead to a different system. The internal spillage rule for a water user $m$ who holds $n_j$ water right class $j$, for $j = 1…J$, and $n'_j$ number of inflow rights class $j$, for $j = 1…J$ is as follows:

$$X_m = \min \left[ \sum_{j=1}^{J} n'_j Y_j + S_m, V \sum_{j=1}^{J} n_j \right] - S_m.$$ 

2. Impacts of various components on the system’s efficiency

Efficiency in allocation of a given quantity of water is attained when there is no possible rearrangement of water use between users and their particular uses or across time (both within a year and between years) that could maintain or increase economic value for all. Achieving efficiency is difficult due to lack of full information. Water has to be partly allocated and used before the actual volume of water that will ultimately be available becomes known. Further, the values of water to particular users are best known by those users and decisions about the timing and quantity of water use ought to best lie with the users. Centrally managed water storage systems are, in principle, likely to be relatively less efficient than decentralized systems in dealing with demand side information. However, externalities of individual water storage and usage decisions, costs of coordination, and the better (relative to individual water users) information that a central authority is likely to have about possible supplies of water may justify centralised management.
2.1 Central water storage and supply of water rights

Central storage rules have important roles in determining the identity of water rights. A water allocation system can have zero central storage, in which case, harvested water inflow is instantaneously allocated to water rights for uses. In this case, the supply of water to water rights may vary wildly and water rights may have low reliability. Central storage can be used to increase the reliability of water rights. To facilitate the discussion on efficient storage in the next section, the relationship between water storage and water supply reliability will first be analysed in this section.

“Ideal” storage reservoir

To see how water storage can change the probability distribution of water allocations, consider an “ideal” dam with an infinite capacity, no storage losses and with the ability (not possible in practice) to borrow water from the future as well as to store. If all the water inflow is instantly allocated, then water allocation has the same distribution as the water inflow. On the other hand, if the excess of water inflow over the expected inflow is stored and only the lesser of the expected water inflow and the total dam content is allocated, then over the long term, the distribution of allocated water is reduced to a deterministic value of the expected water inflow. Storage policy affects the water allocation distribution.

The relationship between water storage policy and the number of water rights in each priority rank can also be visualised using this ideal dam. Initially, suppose that all water inflow is instantaneously allocated and the number of water rights in each priority class is as depicted in Figure 1. The distribution of water allocation is the same as the distribution of water inflow.
FIGURE 1

Consider a case in which the number of water rights in the first class, $N_1$, is to be increased to $\hat{N}_1$ subject to a requirement that the probability distributions of water allocations (including reliability of full allocation) remain unchanged. Without storage, an increase in the number of water rights of class 1 must result in lower reliabilities for all water rights, including class 1. A possible method to maintain the reliability of existing water rights is to store water inflow when it is large and to release water from storage for allocation to the newly issued rights so that the latter receive the same allocation (entirely from stored water) as the allocation received by existing rights in the first category from allocated inflow. The number of additional water rights is constrained by the fact that the expected input to storage must be balanced with the expected release from storage. This water storage/release rule which imitates the water allocation rule applied to existing water rights in class 1 is, however, only one of many alternatives that would allow the number of highest priority rights to be increased while maintaining their reliability. An example of alternative methods that can provide more class 1 water rights with the same reliability (of full allocation) is to release storage water only when inflow is $N_1$ or more and such that class 1 rights then receive full allocation. This alternative method actually maximises the number of new rights (with unchanged reliability) that can be provided from the same expected volume of stored water, but the economic value of a right may not be as high as previously because of lower allocations in years of less-than-full-allocation. Despite its wide acceptance, the usual definition of water supply reliability may not be the most useful from the perspective of economic efficiency.
Realistic dam

In practice, the relationship between water storage and the number of water rights with a given reliability is much more complex when dams have finite capacity and water cannot be borrowed from the future. This relationship is non-linear and dependent on the storage rules adopted. Analysis of the relationship ultimately requires numerical methods. For purposes of demonstrating the non-linear relationship, the storage/release rule used in NSW is assumed. This rule is designed so that at the end of an irrigation season, all the dam reserve (A) is allocated if class 1 water rights are unable to receive the full allocation. When the dam reserve is sufficient to provide full allocation to class 1 water rights for one season, but insufficient for two seasons, then class 1 water rights receive full allocation while class 2 water rights receive no water. When the dam reserve is above twice the level of full allocation for class 1, the excess is allocated to class 2 until full allocation is achieved. When dam reserves are more than sufficient to provide full allocation to all classes, the excess is stored for future allocation \cite{Brennan,2010}. The storage/release rule is depicted in Figure 2 in which \( Y \) denotes aggregate water allocation, \( S \) denotes aggregate water storage, \( K \) denotes the dam capacity, \( N_1 \) denotes the number of class 1 water rights and \( N_2 \) denotes the number of class 2 water rights. Each water right has a nominal volume of 1ML.

FIGURE 2.

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\footnote{The NSW storage rules actually also involve ‘within season’ allocations, under which water is progressively allocated to water rights over the season as the inflows become apparent. Allocations are made using the conservative estimate that the water harvested in the remainder of the season is equal to the minimum historical level. For simplicity, within season allocations are not part of the simulated example. Only the storage/release rule at the end of the season will be analysed.}
For the numerical example, it is assumed that the dam has a capacity of 140ML and that the water inflow to the dam follows a normal distribution with the mean equal to 50 ML and the standard deviation equal to 40 ML. Given the assumed storage/release rule, water allocation follows a Markov process such that water allocation in a period $t$ depends on the allocation in period $t-1$, but not on the allocation in periods before period $t-1$. For any distribution of water allocation in the initial period, the distribution of water allocation in the long run will converge to a unique distribution. To compute this steady state probability distribution of water allocation, water inflow, water storage and water release are first discretised using the same unit (1ML). In the second step, a transition matrix is established to compute the probability distribution of period $t$ water storage ($S_t$) conditional on a given probability distribution of period $t-1$ water storage ($S_{t-1}$). The probability of period $t$ water storage being at a given level can be found by adding the probabilities of all combinations of period $t-1$ water storage and period $t$ water inflow that give rise to that level of period $t$ water storage. Each element of period $t$ water storage probability distribution can then be expressed as a linear function of period $t-1$ water storage probability distribution. Using vector expression, the probability distribution of period $t$ water storage can be written as a product of a transition matrix and the probability distribution of period $t-1$ water storage. Next, the transition matrix is used to calculate the distribution of water storage in period $T$ given the distribution of water storage in a period $t$. When $T$ is large enough, the distribution of water storage in period $T$ is close to the distribution of water storage in the steady state. Based on this steady state distribution of water storage, the distribution of dam reserve and the distribution of water allocation can be computed. Detailed description of the numerical method is provided in the appendix.
When water is not stored for water supply reliability improvement, water allocation is the same as water inflow. The reliability of any water allocation level can be read from the cumulative distribution of water inflow, which is well known for normal distributions. The maximum number of water rights with the full allocation reliability of 95% and a nominal volume of 1ML that can be obtained is zero. When a storage rule is used, the cumulative probability distribution of water allocation can be obtained from the numerical model. Based on this distribution, the reliability of water rights in each class can be determined. For example, when there are only class 1 water rights, the cumulative distribution of water allocation is depicted in Figure 3. A maximum of 41 water rights with 95% reliability can be created.

FIGURE 3

When water is also allocated to class 2 water rights, the number of class 1 water rights with 95% reliability that can be created conditional on the number of class 2 water rights is depicted in Figure 4. Interestingly, the trade-off curve is convex rather than concave as postulated in previous studies. For example, Freebairn and Quiggin [2006] argued that the marginal number of class 2 water rights required for the creation of one class 1 water right should increase as the number of class 1 water rights increases due to the excessive water spillage at the high level of storage required when the number of class 1 water rights is high. However, when the reliability of class 1 water rights is maintained while the reliability of class 2 water rights is not, the trade off is quite different. The number of class 2 water rights needed to form a class 1 water right is lower when the number of class 1 water right is high and the number of class 2 water right is low. This is because a decrease in the number of class 2 water right will reduce the aggregate water allocation and increase the aggregate water storage, which increases the reliability of both class 1 and class 2
water rights and allows to increase the number of class 1 water rights of a given reliability (Figure 4).

FIGURE 4

To provide further insights into changes in the probability distribution of class 2 water rights when water rights are converted between classes, the distribution of water allocation to a class 2 water right when class 2 has 40 water rights is depicted along side the distribution when it has 80 water rights in Figure 5. As can be seen, the two distributions are markedly different. A reduction in the number of water rights in class 2 increases the probability of full allocation at the expense of the probabilities of less than full allocation levels.

FIGURE 5

2.2 Efficiency of the central storage rule

Temporal water allocations by a central authority are mostly based on hydrological information, and typically intended to maintain the reliability of water supply to water rights rather than to maximise the value of water usage. For example, the central storage rule in NSW (mentioned above) is defined based on the total amount of water available at the disposal of the authority and the extreme value of water inflow. It is difficult to construct a central storage rule that maximises the economic value of water use because the central authority does not have private information on the values of water possessed by irrigators. The value of water depends on the level of investment on the farm, the irrigation technology used and crop prices, among other things. These elements vary stochastically over time. The central storage rule expressed solely in terms of the total public water availability, \( S_r(A) \), which is designed to operate optimally at the current time may become suboptimal as stochastic elements change.
For water storage to be optimal (in the sense that it maximises the total expected discounted profit), the storage rule needs to relate to the total water availability in the system being the sum of public water availability and total private storage,

\[ A_T \equiv A + \sum_{m=1}^{M} S_{m,t-1} \], and to the current and forecast irrigated crop prices\(^6\). In practice, irrigated crop prices are often found to follow mean reverting processes with low reversion rates and with these processes, future crop price forecasts are functions of the current crop prices [Deaton and Laroque, 1992]. The optimal central storage rule can be written as a function of total water availability and current irrigated crop prices, \( S^*(A_T, P) \).

In the presence of a central storage rule \( S_c(A) \), each holder \( m, m = 1,...,M \), of a water right \( j \) that has a carryover option will store water according to a rule \( S_m(A_T, S_c(A), P) \) that maximises their expected discounted profit subject to the current total water availability \( A_T \), the central storage rule \( S_c(A) \) and the current irrigated crop prices \( P \). Then the central storage rule \( S_c(A) \) is optimal if the resultant total storage is always equal to the optimal total storage \( S^*(A_T, P) \) defined above:

\[ S_c(A) + \sum_{m=1}^{M} S_m(A_T, S_c(A), P) = S(A_T, P) \].

To be optimal, the central authority needs to know, at the time it formulates the central storage rule, the storage behaviour of private irrigators and how this behaviour is responsive to central storage when the central storage rule is formulated. This is impossible in practice and as a result, central storage rules are unlikely to result in optimal water allocation. Moreover, irrigated crop prices change over time and even if

\(^6\) Other factors such as local rainfall may also affect optimal storage but are not analysed here.
the central storage rule is optimal at the time when it is formed, it may become suboptimal afterwards.

It may be possible to incorporate new information such as crop prices into the central storage rule when markets for water rights exist. As suggested by Freebairn and Quiggin [2006], when crop prices change, the demand for a water right of given sharing priority also changes. Observing new prices of water rights and the rate of conversion between water right classes, the authority could buy low valued water rights and convert them to high valued water rights to sell. Alternatively, water right holders could be allowed to convert their water rights at a given rate. The central storage rule could be adjusted accordingly to reflect the new demanded mix of water rights.

However, as illustrated in Figure 5 for the case of two classes of rights, changing the number of water rights in one class can change the water allocation distribution for water rights in a lower ranking class. Water is more valuable in years when the allocation is less than full, and therefore, a reduction in the probability of receiving less than full allocation water may result in a reduction in the value of the water right, even though the reliability of the water right (probability of receiving full allocation) increases. Even if the conversion results in an increase in the value of class 2 water rights, when the conversion occurs in the other direction, the value of existing class 2 water rights is reduced. Therefore, converting water rights between classes imposes external impacts on other parties.

2.3 Impacts of internal spillage rule

The internal spillage rule as described in section 1.6 limits the usage of storage space by a water user to the space assigned to the user (at least near the accounting time).
The rule is beneficial in that it prevents the negative externality that would occur were one user to store too much water leaving too little space for water harvesting, causing water losses from the system, so affecting other water users. The internal spillage rule can, however, contribute to inefficiency. When there is no external spillage, it is socially optimal to allow water users to store over their assigned storage capacity but such storage space usage is prohibited by the rule. Further, the rule may increase the aggregate amount of water stored in the system (by the amount of confiscated water) and reduce the amount of water available for immediate usage. This will be inefficient in a period of water shortage following a period in which some water users stored near their storage limit.

The inefficiency of the internal spillage rule can be reduced when a rent market is available for storage space. With such a market, users who store over their limit can hire space from users with low storage demand and bypass the internal spillage rule. The inefficiency of the internal spillage rule can also be reduced by measures that reduce transaction costs in the spot water market. When the spot water market is efficient and the transaction cost is low, water trading ensures that no internal spillage occurs. Any storer can reduce the chance of spilling water by selling water to the market for others to store. Storage per water right is equal across users in this perfect world.

Efficiency loss due to the internal spillage rule also depends on the length of accounting period. A long accounting period means that water confiscated via the spillage rule is not available to users for a long period. On the other hand, a short accounting period as in the case of continuous accounting helps to release the water storage quickly and provides more scope for the market to adjust water storage towards the optimal storage rule.
Alternatively, efficient private storage can be encouraged by replacing the internal spillage rule as described with a rule that rations the system’s external (actual) spillage losses to each user according to the amount they store. This alternative rule allows one water right holder to use the dam space unused by other water right holders. The drawback is that the rule does not encourage right holders to take into account the long run cost of storage capacity.

2.4 Impacts of carryover limit

With the internal spillage rule implemented, the negative externality of private storage is ruled out. Allowing users to carry over their unused water provides them an option but not an obligation to defer their water usage to later periods. The option is beneficial to water users and the higher the carryover limit, the larger the option value.

With the central storage decision being made before private storage decisions, water users make their storage decision conditional on the central storage. Higher carryover limits provide water users with greater flexibility in making their storage decisions. When the central storage is higher than the socially optimal level, private storage is zero and carryover limits have no impacts on private storage. However, when the central storage is lower than the socially optimal level, a higher carryover limit allows private storers to increase the aggregate water carryover and reducing the inefficiency of the central storage rule. Certainly, stricter carryover limits cannot reduce the inefficiency of over-storage by the central authority. When carryover limits are at their maximal values and the central storage is zero, all the storage decisions are made by individual water users and the storage rule of the system coincides with the optimal storage rule.
2.5 Impacts of the number of water right classes

Besides allocative efficiency, water allocation systems should also be considered based on their capacity to align water allocation with water demand. Aligning water allocation with water demand helps to reduce the need for water users to participate in the spot market. Based on the profile of water users in the irrigation region, water allocation systems can be designed to minimise the transaction costs.

Water users are strongly heterogeneous in their water demand, but can be broadly classified as flexible irrigators and inflexible irrigators. Flexible irrigators such as those growing annual crops can grow dryland crops when the water price is high, and therefore have elastic water demand. Inflexible irrigators such as perennial croppers often have higher gross margins than flexible irrigators, but cannot replace the perennial crops with other crops when water price is high (due to high establishment costs of perennial crops) and therefore have inelastic water demand. It is likely that inflexible irrigators consume water in all years while flexible irrigators only consume water opportunistically, i.e. when water price is sufficiently low.

When the system has only one class of water rights, water allocation can be adjusted to match the needs of one type of irrigators, but not both. Water trading is frequently required for water users to obtain the level of water they demand. When the system has more than one class of water rights, inflexible irrigators can hold higher priority ranking water rights and flexible irrigators can hold lower priority ranking water rights. Systems with more water right classes are likely to better match water demand of different users. However, as the number of water right classes increases, so does the heterogeneity among rights. This in turn increases buyers’ search costs and other transaction costs. Because markets operate most efficiently when the commodity being allocated is homogenous, Howe et al. [1986] and Young and McColl [2002]
suggest that it may not be optimal to extend the number of water right categories beyond two.

3. Improvements for water allocation systems

The generalised system provided in section 1.6 allows for a wider variety of water rights and water allocation systems than actually in use, at least in Australia. Possible improvements in systems can be considered.

For example, in existing centralised systems with two classes of water rights and a storage/release rule depicted in Figure 2, the carryover limit for each water right can be increased to the nominal volume so that all users can carry over water when they want to (at the risk of losing stored water when the following period allocation is high). The carryover option may be important for holders of class 1 water rights when a drought occurs and it is perceived that the drought may persist into the following year. The internal spillage rule can also be adjusted so that it only applies when the dam spills. In this way, the efficiency of inter-temporal water allocation can be improved.

Similarly, the internal spillage rule can be improved in decentralised systems. Furthermore, existing decentralised systems have only one water right classes and the need to transact in the spot market is likely to be high. It may not be optimal to change the number of water right classes in existing decentralised systems since the cost of doing so may be quite high. However, when consideration is given to convert a centralised system to a decentralised system, a decentralised system with two classes of inflow rights may be considered. Given the limitation of centralised water allocation systems, decentralised systems have been suggested in recent studies [Brennan, 2008; 2010; Brennan and Scoccimarro, 1999] and such suggestions may be
seriously considered in the future. In a decentralised system with two classes of inflow rights, a rule is needed to allocate water inflow to different classes. Since this rule needs to be fixed once and for all time, it should be designed based on the profile of irrigators in the long run. When such profile coincides with the profile of current irrigators, the allocation of water inflow can be similar to the allocation of dam reserve in the centralised system. All the harvested inflow is allocated to class 1 when it is less than a value $X > N_1$. Water is allocated to class 2 when the inflow is greater than $X$ and when the total inflow is greater than $X + N_2$, the excess is allocated to class 1. The value $X$ is fixed so that the probability that class 2 is allocated with water is the same as in the centralised system. In this way, the holders of water inflow class 1 receive water for use when water supply is low and receive water for storage when water supply is abundant. Dam space can be allocated according to usage. Since the holders of water right class 2 only use water opportunistically and do not require water storage, a volume of $N_2$ can be allocated to class 2 and the remaining is allocated to class 1 proportionally. Markets for inflow rights and capacity rights can also be established.

4. Concluding comments

In this paper, water allocation systems used in Australia and elsewhere have been reviewed and a general framework that can be used to analyse each allocation system has been presented. It has been argued that reducing the centralised storage and increasing the scope for private storage in centrally managed water storage systems will increase the inter-temporal water allocation efficiency since individuals continuously adjust their storage rule to incorporate new information. In the limit, this will result in a decentralised system that is most efficient in allocating water between periods. The decentralised systems as currently suggested, however, have only one
class of water right, which is likely to result in a large number of transactions in the spot market. It is suggested that a priority system of inflow rights might be developed within a decentralised system to allow heterogeneous water users to choose the combination of inflow rights that generates a water supply close to their water demand and reduces the need to transact in the spot market. In addition, the internal spillage rule implemented in different systems is likely to result in undesirable water storage and reduce the efficiency of the system. When such efficiency loss is significant, the loss can be reduced by reducing the accounting period or by replacing such rule with a rule that allocate the system’s spillage loss according to water users’ storage.
Figure 1. Water supply reliability and number of water rights
Figure 2. Water storage/release rule used in NSW
Figure 3. Cumulative distribution of water allocation for NSW system with one class of rights.
Figure 4. Trade-off of the numbers of water rights in two classes.
Figure 5. Distribution of water allocation to a class 2 water right under different numbers of class 2 water rights.
Appendix: Numerical method to compute the stationary distribution of water storage

In this appendix, the method developed by Moran [1954a; 1954b; 1959] will be used to calculate the stationary distribution of water allocation given the NSW central storage rule. There are $N_1$ class 1 water rights and $N_2$ class 2 water rights, each has a nominal volume of 1ML. It is assumed that water is allocated only at the end of the season. The central storage rule is to store the excess over $N_1$ when the available water $A_t$, defined as the sum of the carryover from the previous period and the water inflow to the dam in the current period, is above $N_1$ and less than $2N_1$, store only $N_1$ when the available water is above $2N_1$ and less than $2N_1 + N_2$, store $N_1$ and all the excess of available water over $2N_1 + N_2$ when the available water is above $2N_1 + N_2$. An available water exceeding the dam capacity $K$ will cause overflow. This storage rule and the resulted release rule are depicted in Figure 2.

Given the storage rule, water storage follows a stationary distribution in the steady state, i.e. in the steady state, $S_{t-1}$ and $S_t$ will have the same probability distribution. As explained in the Section 2.1, this distribution can be calculated numerically in three steps: discretising the variables, establishing a transition matrix linking water storage distributions in successive periods and simulating for a sufficient number of periods.

Let $X_t$, the water inflow in period $t$, take values 0, 1, 2, … with probabilities $p_0, p_1, p_2, …; S_{t-1}$ take values 0, 1, …, $M$ with probabilities $P = (P_0, P_1, ..., P_M)$ and $S_t$ take values 0, 1, …, $M$ with probabilities $P' = (P'_0, P'_1, ..., P'_M)$, where $M$ is the maximal level of water storage (achieved when the dam is full). The maximal storage $M$ and hence the steady state distribution of water storage depends on the level of dam capacity $K$ relative to $N_1, 2N_1$ and $2N_1+N_2$ (Figure 2). If $K$ is less than $N_1$, no water is stored in any period and computation is not necessary for steady state water storage.
distribution. This leaves three cases: \( K > 2N_1 + N_2,\ 2N_1 < K < 2N_1 + N_2, \) and \( N_1 < K < 2N_1. \)

**Case 1:** \( K > 2N_1 + N_2 \)

The probability that a level of water storage \( S_t \) occurs in period \( t \) is found by aggregating the probabilities of all combinations \((S_{t-1}, X_t)\) giving rise to such level. To find these combinations, the levels of water inflow in period \( t, X_t \), leading to \( S_t \) are listed for each previous period storage level \( S_{t-1} \). When several levels of water inflow \( X_t \) can lead to the given level of water storage in period \( t \), for a given \( S_{t-1} \), a mass point in the probability distribution of \( S_t \) is obtained at this storage level. For the current case, the distribution of \( S_t \) has three mass points at \( S_t = 0 \), \( S_t = N_1 \) and \( S_t = K - N_1 - N_2 \). The probability distribution of \( S_t \) can be found by calculating the probability that each level of \( S_t \) occurs using the following equations:

\[
P_0 = P_0 \sum_{i=0}^{N_1} p_i + P_1 \sum_{i=0}^{N_1-1} p_i + ... + P_{N_1} p_0
\]

\[
P_1 = P_0 p_{N_1 + 1} + P_1 p_{N_1} + ... + P_{N_1} p_1 + P_{N_1 + 1} p_0
\]

\[...\]

\[
P_{N_1} = P_0 \sum_{i=2N_1}^{2N_1 + N_2} p_i + P_1 \sum_{i=2N_1 - 1}^{2N_1 + N_2 - 1} p_i + ... + P_{N_1} \sum_{i=2N_1}^{N_2} p_i + P_{N_1 + 1} \sum_{i=0}^{N_2} p_i + ... + P_{2N_1 + N_2} p_0
\]

\[
P_{N_1 + 1} = P_0 p_{2N_1 + N_2 + 1} + P_1 p_{2N_1 + N_2} + ... + P_{2N_1 + N_2 + 1} p_0
\]

\[...\]
\[ P_{K-N_1-N_2} = P_0 \sum_{i=K}^{\infty} p_i + P_1 \sum_{i=K-1}^{\infty} p_i + \ldots + P_{K-N_1-N_2} \sum_{i=N_1+N_2}^{\infty} p_i \]

**Case 2:** \( 2N_1 < K < 2N_1 + N_2 \)

In this case, the maximum storage is \( M = N_1 \). The probability of each level of \( S_i \) that is below \( N_i \) can be calculated using the first \( N_1 \) equations listed in Case 1. The probability that \( S_i \) is equal to \( N_1 \) is:

\[ P_{N_1} = P_0 \sum_{i=2N_1}^{\infty} p_i + P_1 \sum_{i=2N_1-1}^{\infty} p_i + \ldots + P_{N_1} \sum_{i=N_1}^{\infty} p_i \]

**Case 3:** \( N_1 < K < 2N_1 \)

In this case, the maximum storage is \( M = K - N_1 \). The probability of each level of \( S_i \) that is below \( K - N_1 \) can be calculated using the first \( K - N_1 \) equations listed in Case 1. The probability that \( S_i \) is equal to \( K - N_1 \) is:

\[ P_{K-N_1} = P_0 \sum_{i=K}^{\infty} p_i + P_1 \sum_{i=K-1}^{\infty} p_i + \ldots + P_{K-N_1} \sum_{i=N_1}^{\infty} p_i \]

The above equation systems can be written using matrix notations as \( \mathbf{P} = \mathbf{BP} \), where \( \mathbf{B} \) is a \((M+1) \times (M+1)\) matrix with its elements being the coefficients of \( P_0, P_1, \ldots, P_M \). After two periods, the distribution of water storage is \( \mathbf{P}^2 = \mathbf{BP} = \mathbf{BBP} = \mathbf{B^2P} \). By calculating the \( n \) power of matrix \( \mathbf{B} \) and let \( \mathbf{P} \) be a vector with all zero, except for one element which takes value 1, the stationary distribution of water storage can be obtained.

Given the steady state distribution of water storage \( S \), the distribution of water availability \( A \) can be found as follows. Let \( \mathbf{R} = (R_0, R_1, \ldots, R_{K+1}) \), where \( s \) is a positive
large number, be the probability that \( A \) takes values \((0,1,\ldots, K+s)\). Let 
\[
\mathbf{P} = (\mathbf{\overline{P}_0}, \mathbf{\overline{P}_1}, \ldots, \mathbf{\overline{P}_M})
\]
be the stationary distribution of water storage. Then:
\[
R_0 = \mathbf{\overline{P}_0} p_0
\]
\[
R_1 = \mathbf{\overline{P}_0} p_1 + \mathbf{\overline{P}_1} p_0
\]
\[
\ldots
\]
\[
R_{K+s} = \mathbf{\overline{P}_0} p_{K+s} + \ldots + \mathbf{\overline{P}_M} p_{K+s-M}
\]
The distribution of available water can then be used to calculate the distribution of water allocation for each of the three cases listed above. Let 
\[
\mathbf{Q} = (Q_0, Q_1, \ldots, Q_U)
\]
be the distribution of water allocation, where \( U \) is the maximum level of water allocation. In Case 1, where the maximum allocation level is \( U = N_1 + N_2 \), the probability of each water allocation level can be calculated as:
\[
Q_0 = R_0
\]
\[
Q_1 = R_1
\]
\[
\ldots
\]
\[
Q_{N_1} = \sum_{i=N_1}^{2N_1} R_i
\]
\[
\ldots
\]
\[
Q_{N_1 + 1} = R_{2N_1 + 1}
\]
\[
\ldots
\]
\[
Q_{N_1 + N_2} = \sum_{i=2N_1 + N_2}^{K+s} R_i
\].
In Case 2, the maximum level of water allocation is $U = K - N_i$. The probability that water allocation attains a level below $K - N_i$ can be calculated using the first $K - N_i$ equations listed in Case 1. The probability that water allocation is equal to $K - N_i$ is given by:

$$Q_{N_i + N_2} = \sum_{i=K-N_i+1}^{K+N} R_i.$$

In Case 3, the maximum level of water allocation is $U = N_i$. The probability that water allocation attains a level below $N_i$ can be calculated using the first $N_i$ equations listed in Case 1. The probability that water allocation is equal to $N_i$ is given by:

$$Q_{N_i + N_2} = \sum_{i=N_i+1}^{K+N} R_i.$$
References


Young, M. D., and J. C. McColl (2002), Robust Separation: A search for a generic framework to simplify registration and trading of interests in natural resources. CSIRO Land and Water.